The Allocation of Talent
and U.S. Economic Growth

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Abstract

In 1960, 94 percent of doctors and lawyers were white men. By 2010, the fraction was just 62 percent. Similar changes in other highly-skilled occupations have occurred throughout the U.S. economy during the last fifty years. Given that innate talent for these professions is unlikely to differ across groups, the occupational distribution in 1960 suggests that a substantial pool of innately talented black men, black women, and white women were not pursuing their comparative advantage. This paper examines the macroeconomic consequences of the remarkable convergence in the occupational distribution between 1960 and 2010 through the prism of a Roy model. We find that about one-third of growth in aggregate output per worker over this period may be explained by the improved allocation of talent, mostly from falling barriers to accumulating human capital.

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1. Introduction

Over the last 50 years, there has been a remarkable convergence in the occupational distribution between white men, women, and blacks. For example, in 1960, 94 percent of doctors and lawyers were white men. By 2010, the fraction was just over 60 percent.¹

Similar changes occurred throughout the economy during the last fifty years, particularly among highly-skilled occupations. A very large literature attempts to explain why white men differ in their occupational distribution relative to women and blacks and why those differences have been changing over time.² Yet no formal study has assessed the effect of these changes on aggregate productivity. Given that innate talent for many professions is unlikely to differ across groups, the occupational distribution in 1960 suggests that a substantial pool of innately talented blacks and women were not pursuing their comparative advantage. The resulting (mis)allocation of talent could potentially have important effects on aggregate productivity.³

This paper measures the aggregate productivity effects from the changing allocation of talent for women and blacks from 1960 to 2010. To do this, we examine the differences in labor market outcomes between race and gender groups through the prism of a Roy (1951) model of occupational choice. While it is not our goal to causally identify any specific factor that explains differences in occupational sorting, our model is broad enough to encompass many of the common explanations highlighted in the literature. For example, our model allows for race-sex groups to differ in (1) the extent of labor market discrimination they face in each occupation, (2) the extent to which they face frictions in the ability to accumulate occupations specific human capital, (3) occupational preferences and (4) their productivity/preference for home production. Using a variety of empirical moments, along with our modeling structure, we develop a procedure to separately identify the contribution to U.S. productivity growth of changes in each of these factors over time. Across our various specifications, we find that roughly one-third of the growth in U.S. GDP per person between 1960 and

¹These statistics are based on the 1960 Census and the 2010-2012 American Community Surveys. We discuss the sample in more detail below.
²See, for example, Blau (1998), Blau, Brummund and Liu (2013b), Goldin (1990), Goldin and Katz (2012), Smith and Welch (1989) and Pan (2015). Detailed surveys of this literature can be found in Altonji and Blank (1999), Bertrand (2011), and Blau, Ferber and Winkler (2013a). ³Hsieh and Klenow (2009) suggest that differences in barriers that affect the allocation of inputs can explain some of the differences in productivity across countries.
2010 can be explained by declining frictions facing white women, black men and black
women. Falling barriers to accumulating human capital played the largest role, with a
more modest role for declining labor market discrimination. The declining frictions
to human capital accumulation and declining labor market discrimination resulted
in women and blacks sorting to occupations that allow them to better exploit their
comparative advantage.

The paper proceeds in four parts. We begin the paper by developing our model
of occupational choice and aggregate production. We assume that efficiency units
of labor is the only input into aggregate production. Each individual faces a multi-
period lifecycle. In the pre-period, individuals make initial human capital decisions
which they carry with them over the remaining periods of their life. In each subse-
quent period, workers decide whether to work in the market sector or the home sector.
Conditional on choosing to work in the market sector, individuals choose a market oc-
cupation. We assume that every person is born with a range of talents across all possible
occupations and chooses the occupation with the highest expected utility. In our base
specification, we assume that the distribution of talent across market occupations is
similar across sex-race groups. This implies that the likelihood that a woman or black
man receives a high talent draw in a market occupation (e.g., doctors, lawyers, nurses)
is the same as that for a white man. We model the aggregate demand for skills within
each occupation by allowing each occupation to receive its own productivity draw. To
close the model, the supply and demand for skills in each occupation will determine
the price of skills in that occupation. Aggregate output is just a CES aggregate of the
production in each occupation.

As alluded to above, we allow for four other forces within the model that results
in the occupational distribution to differ across race-sex groups. First, the model
incorporates discrimination within the labor market. We model labor market discrim-
ination as an occupation specific wedge between the wages of white women, black men

\footnote{However, in some specifications, we build on the work of Rendall (2010) and allow for the possibility
that men have a absolute advantage in brawn-intensive occupations (such as construction). This implies
that changes in the extent to which occupations become less brawny or changes in the returns to brawn-
vs. brain-intensive occupations can also explain some of the changes in the occupational sorting between
men and women over time. As we show below, essentially all of our aggregate gains come from the fact
that women and blacks moved into high skilled occupations like doctors, lawyers, executives, etc. Our
maintained assumption throughout the paper is that all groups draw from the same distribution of innate
talent in these high skilled occupations.}
and black women relative to white men. This “tax” is a proxy for many common formulations of taste-based and statistical discrimination found in the literature.\textsuperscript{5} We show that occupations that discriminate in the labor market towards a given group will have a lower fraction of that group working in that occupation. Second, our model allows for differential frictions in human capital investments across groups and occupations. We model these frictions as increased monetary costs associated with accumulating occupation specific human capital. These costs are a proxy for many different race and gender specific factors. For example, this could proxy for parental and teacher discrimination in favor of boys in development of certain skills, historical restrictions on the admission of women to colleges or training programs, differences in school quality between Black and White neighborhoods, and social norms that steer groups away from certain occupations.\textsuperscript{6} Third, we allow for occupational preferences to differ across groups. Such differences in occupational preferences between men and women have been highlighted in the work of, among others, Johnson and Stafford (1998), Altonji and Blank (1999), and Bertrand (2011). Finally, the model allows for group specific differences in the productivity within the home sector. While we model this factor as productivity differences in the home sector, it is isomorphic to preferences for the home sector. In this sense, the model also captures changes in social norms for women working at home and time series changes in fertility.\textsuperscript{7}

In the second part of the paper, we show how we can use the model structure and available data to tease out the factors that drive differential occupational sorting between race-sex groups. All four of our potential mechanisms can affect the occ-

\textsuperscript{5}See, for example, Becker (1957), Phelps (1972) and Arrow (1973). A summary of such theories can be found in Altonji and Blank (1999).


\textsuperscript{7}The literature on changes in female labor supply due to changes in productivity, preferences, and social norms is extensive. See, for example, Fernández, Fogli and Olivetti (2004a) and Fernández (2013) for a discussion of the role of cultural forces in changing female labor supply, Greenwood, Seshadri and Yorukoglu (2005) for the role of home durables in explaining changes in female labor supply, and Goldin and Katz (2002) for the role of the birth control on changing female labor supply. Surveys of much of this literature can be found in Costas (2000) and Blau, Ferber and Winkler (2013a).
pational distribution of different groups relative to white men. However, given our distributional assumptions on talent draws, the different forces have different implications for relative wage gaps across occupations. By focusing on the occupational choice across market occupations for individuals early in their lifecycle, we can use both quantity and wage data to distinguish “composite frictions” from preferences. Our composite friction measure is a combination of labor market discrimination and barriers to human capital attainment.

To facilitate identification, we assume that occupational talent is drawn from an extreme value distribution. This assumption decouples the share of a given race-sex group working in a given occupation (relative to white men) from the average wages of a given race-sex group working in that occupation (relative to white men) when a composite friction is present in that occupation. The reason for this is that when a composite friction is present in a given occupation, there are two offsetting forces with respect to average wages in that occupation. The composite friction acts as a tax on the group’s wages reducing their return to working in that occupation. However, the sorting model implies that only really talented members of the group would then enter that occupation. The increased quality of workers in the occupation results in higher average wages for that group in the occupation. With our extreme value assumption, these two forces exactly offset implying that wage gaps between a group and white men are constant across occupations even though frictions could differ across occupations. Preference differences, on the other hand, do not have this implication. If a group systematically does not like a given occupation, wages will have to be higher for that group in the occupation to draw them in to the occupation. So, examining wage gaps across market occupations (relative to white men) for younger workers allows us to distinguish preferences from composite frictions. Further more, our model yields an expression showing that conditional on the relative wage gaps between groups, the relative propensity to work in a market occupation between two groups early in their lifecycle pins down a unique expression for the composite friction these individuals face in that occupation.

The next step of our procedure is to distinguish among the two parts of the composite friction. Is the composite friction being driven by labor market discrimination or is it drive by barriers to human capital accumulation? To tease these two fac-
tors apart from each other, we exploit the lifecycle structure of our model. Given our model assumptions, workers make a bulk of their human capital decisions when young. Likewise, we assume occupational preferences are constant over a worker’s lifetime. On the other hand, workers can choose occupations in every period of their life. In terms of identification, the human capital frictions are something akin to a cohort effect in wages while the labor market discrimination is something akin to a time effect in wages. We assume that absent these frictions, the wage-experience profile of a given group would be identical to that of white men. Collectively, using wage and quantity decisions of different cohorts as they progress through their lifecycle allows us to decompose the composite frictions into the human capital barriers and labor market discrimination. Finally, we can use observed labor market participation data, the model structure, and our estimates for preferences, labor market discrimination, and human capital barriers to estimate the productivity/preferences for the home sector for each group.

In the third part of the paper, we use the full structure of our model to jointly estimate all the remaining parameters of the model. This includes the occupation specific productivities and the returns to skill within each occupation. These parameters are pinned down by the size of the occupation (across all groups) and the average wages in the occupation (across all groups) and do not differ across groups. We estimate our model separately every decade using either Census data (1960-2000) or data from the pooled American Community Survey (2010-2012). With the fully estimated model, we can then perform counterfactuals as to how the aggregate economy would have evolved had factors affecting the relative occupational sorting across groups remained constant over time. Our results suggest that roughly 30 percent of GDP per person and 35 percent of GDP per worker can be explained jointly by declines in barriers to human capital and declines in labor market discrimination for white women, black men and black women between 1960 and 2010. The overwhelming majority of that decline can attributed to declines in barriers to human capital attainment. We also estimate that changing occupational preferences for women and black men did not contribute to changes in U.S. productivity over the last half-century.

Given our model, we can perform a variety of additional counterfactuals. First, the majority of the productivity gains we estimate come from changes to white women.
This stems from the fact that white women are a much larger share of the population relative to black men and black women. Additionally, we estimate that white men did not benefit at all from the declines in human capital barriers and labor market discrimination that occurred between 1960 and 2010. When these frictions were binding, marginally talented men filled the void in these high skilled occupations providing some additional gains to white men. Our estimates also imply that the declining human capital barriers and labor market discrimination only explain jointly about 12 percent of the aggregate change in labor force participation. We estimate that changes in productivity/preferences in the home sector for women explain the bulk of the change in labor market participation. Finally, most of the gains we estimated from declining discrimination and barriers to human capital attainment occurred prior to 1990. As a result, part of the reason the U.S. may have experienced lower growth during the 1990s and 2000s is attributed to the fact that white women, black men, and black women have converged to white men. This does not mean that the occupational distributions of the different group are the same or that the wage gaps between the groups are zero. It does mean, however, that there has not been much relative movements in these differences since 1990.

In the final part of the paper, we use external data to assess the plausibility of our model. For example, we use our model to compute a composite friction measure for black men for each U.S. state pooling together data from the 1980 and 1990 census. We then correlate our composite friction measure with survey based state measures of racial discrimination as calculated by Charles and Guryan (2008). Our model based estimates match up very closely with Charles and Guryan’s survey based measures suggesting that our model estimates are capturing potential labor market frictions highlighted by other researchers using other datasets. Additionally, we compare our model based estimates of female labor supply elasticities for different cohorts to empirical estimates of female labor supply elasticities for these same cohorts as reported in Blau and Kahn (2007). Again, our model matches well the decline in labor supply elasticities for more recent cohorts as reported in Blau and Khan. Collectively, we view the fact that we can replicate measures of discrimination and labor supply elasticities from other authors – even though such moments were not used to discipline our model – as being evidence of our model capturing salient features of the U.S. labor market during this
Finally, we realize that there is a very large literature examining changes in labor market outcomes across race-sex groups within the U.S. over the last five decades. Much of this literature explores a very particular mechanism that explains these changing labor market conditions. Our goal in this paper is slightly different. We want to ask how these broad forces highlighted in the literature affect macroeconomic aggregates.\footnote{Several additional recent papers are worth noting for related contributions. Ellison and Swanson (2010) show that high-achieving girls in elite mathematical competitions are more geographically concentrated than high-achieving boys, suggesting many girls with the ability to reach these elite levels are not doing so. Cavalcanti and Tavares (2007) use differences in wage gaps across countries in a macro model to measure the overall costs of gender discrimination and find that it is large. Albanesi and Olivetti (2009) study the gender earnings gap in a model of home production, while Dupuy (2012) studies the evolution of gender gaps in world record performances in sport. Beaudry and Lewis (2012), looking across cities, suggest that much of the change in the gender wage gap can be explained by a change in the relative price of skills.}

2. Model

The economy consists of a continuum of workers in $M$ market occupations or in the home sector. Workers are indexed by occupation $i$, group $g$ (such as race and gender), and cohort $c$. Each worker possesses heterogeneous abilities — some people are good economists while others are good nurses. The basic allocation to be determined in this economy is how to match workers with occupations.

2.1. Firms

A representative firm produces final output $Y$ from workers in $M$ occupations:

$$Y = \left[ \sum_i \left( A_i \cdot \sum_g H_{ig} \right) \right]^\frac{\sigma - 1}{\sigma}$$

(1)

where $H_{ig}$ denotes the total efficiency units of labor provided by group $g$ in occupation $i$ and $A_i$ is the exogenously-given productivity of occupation $i$. The parameter $\sigma$ represents the elasticity of substitution across occupational output in aggregate production.

Following Becker (1957), we assume the owner of the firm in the final goods sector discriminates against workers of certain groups. We model the “taste” for discrimination as lower utility of the owner when she employs workers from groups she dislikes.
Her utility is given by

\[ U_{owner} = Y - \sum_i \sum_g w_{ig} \cdot H_{ig} - \sum_i \sum_g \tau_{ig}^w \cdot w_{ig} \cdot H_{ig} \]  \hspace{1cm} (2)

where \( w_{ig} \) denotes the wage per unit of efficiency unit of labor in occupation \( i \) paid to workers from group \( g \). Note that \( w_{ig} \) potentially differs across groups in the same occupation. This is because the owner is prejudiced and suffers a utility loss denoted by \( \tau_{ig}^w w_{ig} \) from employing workers from group \( g \) in occupation \( i \). The first two terms in Equation (2) denotes the firm’s profit and the last term is the total utility loss of the firm’s owner from hiring workers she dislikes. If we assume \( \tau^w = 0 \) for white men (for all occupations), then \( w_{ig} = (1 - \tau_{ig}^w) \cdot w_i \) where \( w_i \) is the wage per efficiency unit of labor of white men in occupation \( i \). Intuitively, when the owner hires a worker from a group she dislikes, her utility loss is compensated by the lower wage she pays to these workers.

A second firm (a “school”) sells educational goods \( e \) to workers who use it as an input in their human capital.\(^9\) We assume the school’s owner dislikes providing \( e \) to certain groups. The utility of the school’s owner is

\[ U_{school} = \sum_i \sum_g (R_{ig} - 1) \cdot e_{ig} - \sum_i \sum_g \tau_{ig}^h R_{ig} e_{ig} \]  \hspace{1cm} (3)

where \( e_{ig} \) denotes educational resources provided to workers from group \( g \) in market sector \( i \), \( R_{ig} \) denotes the price of \( e_{ig} \), and \( \tau_{ig}^h \) represents the owner’s distaste from providing educational resources to workers from group \( g \) in sector \( i \). We think of \( \tau_{ig}^h \) as a shorthand for complex forces such as discrimination against blacks or women in admission to universities, or differential allocation of resources to public schools attended by black vs. white children, or differential parental investments made toward building up math and science skills in boys relative to girls.

The key thing to note is that the price of \( e \) varies across groups. If we normalize \( \tau_{ig}^h \) of white men to zero, then \( R = 1 \) for white men and \( R_{ig} = 1 + \tau_{ig}^h \). Groups that are discriminated in the provision of human capital pay a higher price for \( e \), and the higher price compensates the school owner for her disutility.

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\(^9\)We describe the human capital accumulation process below.
In sum, discrimination in the human capital sector increases the cost of acquiring human capital for members of the discriminated group. Discrimination in the labor market lowers the wage per efficiency unit for workers in the discriminated group. We will next see how these forces affect group occupations and group wages.

2.2. Workers

As in a standard Roy (1951) model of occupational choice, workers are endowed with idiosyncratic talent $\epsilon$ in each market occupation as well as a separate idiosyncratic talent $\epsilon_{\text{home}}$ in the home sector. We add to this standard framework our notion of discrimination in the labor market and frictions in human capital accumulation described above. We also allow for group specific preferences for an occupation that are not driven by labor market discrimination or frictions in human capital attainment. For example, there might have been widely held social reprobation towards female doctors or lawyers in the past which discouraged women from choosing these occupations. Social reprobation could be highly correlated with labor market discrimination and frictions in human capital accumulation, but we later show how to differentiate between preferences and these other forces.

We add the following additional features to this setup. First, we assume that individuals invest in their human capital and choose a market occupation in the first period, after which they work for three periods (young, middle, old). We assume that invested human capital and market occupation are fixed after the choice is made. Second, we assume individuals choose between the home sector or their chosen market occupation each period (but we do not allow people to change market occupations). We introduce these features to capture the idea that some frictions may only affect certain cohorts of a given group at all points in time while other frictions affect all cohorts of the group at the same point in time. As we discuss later, we lean on this distinction to differentiate between whether frictions in initial human capital choice or discrimination in the labor market is driving the occupational choice differences across groups.

We model group specific occupational preferences as directly affecting the utility of individuals of a group that choose a given market occupation. Specifically, for a worker
of group $g$ from cohort $c$ that chooses market occupation $i$, lifetime utility $U$ is given by

$$
\log U = \beta \sum_{t=c}^{c+2} \log C(c, t) + \log (1 - s(c)) + \log z_{ig}(c)
$$

(4)

where $C(c, t)$ is consumption for individuals of cohort $c$ in period $t$ of their lifecycle, $1 - s$ is leisure time during the pre-period where human capital investments are made, and $z_{ig}$ denotes the utility cost of working in occupation $i$ among members of group $g$.\footnote{The first time period when an individual from cohort $c$ can work is period $c$.} Social disapproval lowers utility holding consumption fixed. We will omit subscripts on other individual-specific variables (such as $s$ and $C$ in this case) to keep the notation clean. However, $z_{ig}$ does have subscripts to emphasize that it varies across groups and occupations. $\beta$ parameterizes the tradeoff between consumption and time spent accumulating human capital.

A few comments on our utility specification. First, in the initial period, individuals are endowed with a unit of time. The tradeoff that individuals make in this period is how much time to allocate to schooling ($s$) versus leisure ($1 - s$). Second, in three working periods of their lifecycle, individuals receive utility from consumption. We abstract from discounting for simplicity. Finally, we assume that individuals who choose market occupation $i$ pays the utility cost $z_{ig}$. Conditional on choosing an occupation, household utility is determined by the time they spend accumulating occupation specific human capital in the initial period, the disutility associated with having chosen to specialize in that occupation, and the consumption funded in each period of the working life with occupation specific earnings (or home production).

As noted above, individuals acquire human capital $h$ in the initial period, and this human capital remains fixed over their lifetime. We do not allow workers to return to school when they are older, but this is rare in the data. Individuals use both time $s$ and goods $e$ in the production of $h$ such that:

$$
h = s^\theta e^\eta
$$

(5)

In terms of timing, we temporally separate the individual’s decision to accumulate human capital from the individual’s decision to work in the market versus home sector. In particular, we assume that individuals will work in their chosen market sector when
making their human capital decisions. Once they have accumulated their occupation
specific human capital, the individual then decides whether to allocate their labor sup-
ply to working in the market sector (in their chosen occupation) or in the home sector.
While this assumption may seem stark, it may not be grossly at odds with the data. For
example, in the 1960s and 1970s many women went to college and accumulated human
capital only to later decide whether to work in the home sector. These decisions were
usually made after getting married and having children.

Household consumption can be generated either by allocating time to the market
during working years and purchasing consumption or by allocating time to the non-
market sector and home producing consumption. To start, we will focus on consump-
tion for individuals who work in the market sector.

Specifically, if a worker chooses occupation $i$ and works in the market sector, con-
sumption in each period of their lifecycle is market income minus a portion of past
expenditures on education. Specifically, consumption of cohort $c$ at time $t$ is given by:

$$ C(c, t) = (1 - \tau^w(c, t)) w(t) e T h - e(c, t)(1 + \tau^h(c)). $$

(6)

$e(c, t)(1 + \tau^h(c))$ are payments in period $t$ for educational expenditures undertaken in
the initial period, where the lifetime budget constraint is $e(c) = \sum_{t=c}^{c+2} e(c, t)$. Individuals
borrow $e(c)(1 + \tau^h(c))$ in the first period to purchase $e(c)$ units of human capital, which
is then repaid over their lifetime. The labor market distortion $\tau^w$ lowers individual
income for a given amount of efficiency units of labor. Individual income is the product
of the price for efficiency unit of skill $w$, the idiosyncratic talent draw in the worker’s
chosen occupation $\epsilon$, the efficiency of human capital $T$, and the individual’s acquired
human capital $h$. We will interpret $T$ as the returns to experience of a given cohort so $T$
would increase simply as a cohort ages. However, an alternative interpretation is that $T$
reflects differences in innate skill across groups in a given occupation. For example, in
some occupations, brawn may be a desirable attribute. If men are physically stronger
than women on average, than one would expect more men in occupations requiring
more physical strength, such as firefighting or construction. In our base specification,
we assume that the mean level of innate skill is constant across all groups. However, in
some of our robustness specifications, we will relax this assumption by allowing men
to have an absolute advantage in “brawny” occupations implying they have a higher $T$
in those occupations.

Discrimination in the labor market directly lowers the wage per efficiency unit paid to workers of the discriminated group. The discrimination shows up as a “tax” on individual earnings. Given our assumption that the firm owner discriminates against all workers of a given group, $\tau_w$ affects all the cohorts of group $g$ equally at a given point in time. Likewise, frictions in human capital attainment, $\tau_h$, affect consumption directly by increasing the cost of $e$ in Equation (6). Also, these frictions will indirectly affect consumption by lowering the amount of the individual’s acquired human capital $h$. Because the human capital decision is only made once and fixed thereafter, $\tau_h$ for a given occupation varies across cohorts and groups, but is fixed for a given cohort-group over time.

Under these assumptions, given an occupational choice, the occupational wage $w_i$, and idiosyncratic ability $\epsilon$ in the occupation, the individual from cohort $c$ chooses $C$ in each period of working life and $e$ and $s$ in the initial period to maximize lifetime utility given by (4) subject to the constraints given by (6) and $e(c) = \sum_{t=c}^{c+2} e(c, t)$. Individuals will choose the time path of $e(c, t)$ such that expected consumption is constant and equal is one third of expected lifetime income. In turn, lifetime income depends on expected value of $w_i$, $\tau_{ig}^w$, and $T$ in the three periods. To obtain additional results, we assume that individuals anticipate that their experience (as represented by $T$) will grow with age but that the $\tau_{ig}^w$ and $w_i$ they observe when young is their best guess for future levels of these two variables. The amount of time and goods an individual spends on human capital are then given by:

$$s_i^* = \frac{1}{1 + \frac{1-\eta}{3\phi_i}}$$

$$e_{ig}^* = \left(\frac{\eta(1-\tau_{ig}^w)w_iT^s_i\phi_i}{1 + \tau_{ig}^h}\right)^{\frac{1}{1-\eta}}$$

where $T = 1 + T_m + T_o$ and $T_m$ and $T_o$ denote the returns to experience in middle age and when old. Time spent accumulating human capital is increasing in $\phi_i$. Individuals in high $\phi_i$ occupations acquire more schooling and have higher wages as compensation for time spent on schooling. Forces such as $w_i$, $\tau_{ig}^h$, and $\tau_{ig}^h$ do not affect $s$ because they have the same effect on the return and on the cost of time. In contrast, these forces
change the returns of investment in *goods* in human capital (relative to the cost) with an elasticity that is increasing in $\eta$. These expressions hint at why we use both time and goods in the production of human capital. Goods are needed so that distortions to human capital accumulation matter. As we show below, time is needed to explain average wage differences across occupations.

After substituting the expression for human capital into the utility function, indirect utility from working in occupation $i$ when young is

$$U_{worker}^* = \left( \frac{w_i s_i^\phi_i (T/3) \left( (1 - s_i) z_{ig} \right)^{1-\eta} \epsilon_i \eta'(1 - \eta)^{1-\eta}}{\tau_{ig}} \right)^{\frac{3\beta}{1-\eta}} \tag{7}$$

Here, $\tau_{ig}$ is a “composite” distortion that summarizes the effect of labor market discrimination and human capital frictions:

$$\tau_{ig} \equiv \frac{(1 + \tau_{hig})^\eta}{1 - \tau_{wig}}. \tag{8}$$

More human capital frictions or labor market discrimination increase $\tau_{ig}$, which lowers indirect utility from choosing occupation $i$. Similarly, lower preferences for members of group $g$ in occupation $i$ is represented as a low value of $z_{ig}$, which also lowers indirect utility for members of the group in the occupation.

After the individual chooses a market occupation, she then decides whether to work in the chosen market sector or in the home sector. If the individual chooses the home sector, consumption is given by:

$$C(c, t) = w^{home}(t) \epsilon^{home} T^{home}(t) h - e(c, t)(1 + \tau^h(c)). \tag{9}$$

Income in the home sector is the product of the wage per efficiency unit of home talent $w^{home}$, the idiosyncratic talent draw $\epsilon^{home}$ in the home sector, efficiency of human capital in the home sector $T^{home}$, and accumulated human capital $h$. We assume $w^{home}$ varies over time but not across groups. For example, technological innovations in the home sector emphasized by Greenwood, Seshadri and Yorukoglu (2005) can be viewed as a decline in $w^{home}$ that affects men and women equally. $T^{home}$, however, potentially varies across groups (thus the subscript $g$) and time. For example, innovations in
contraception emphasized by Goldin and Katz (2002) can be thought of as as changes in $T_{g}^{\text{home}}$ of women.\textsuperscript{11} Finally note the assumption that the value of acquired human capital $h$ in the home sector is the same as in the market sector. In addition, we assume that the payment for human capital $e(c, t)(1 + \tau^{h}(c))$ is the same regardless of whether the individual chooses to work in the home or in the market sector.

Finally, turning to the distribution of the idiosyncratic talent, we borrow from McFadden (1974) and Eaton and Kortum (2002). Each person gets a skill draw $\epsilon_{i}$ in each occupation. Talent in the market occupations are drawn from a multivariate Fréchet distribution:

$$F_{g}(\epsilon_{1}, \ldots, \epsilon_{M}) = \exp \left[ - \sum_{i=1}^{M} \epsilon_{i}^{-\theta} \right]. \quad (10)$$

The parameter $\theta$ governs the dispersion of skills, with a higher value of $\theta$ corresponding to smaller dispersion. Similarly, we assume talent in the home sector $\epsilon_{\text{home}}$ is drawn from a Fréchet distribution with the same dispersion parameter $\theta$. $M$ is the number of market occupations over which individuals get talent draws.

### 2.3. Occupational choice

Given the above assumptions, the occupational choice problem thus reduces to picking the occupation that delivers the highest value of $U_{ig}^{*}$ in the first period. Because talent is drawn from an extreme value distribution, the highest utility can also be characterized by an extreme value distribution, a result reminiscent of McFadden (1974). The overall occupational share can then be obtained by aggregating the optimal choice across people, as we show in the next proposition.\textsuperscript{12}

**Proposition 1** (Occupational Choice): Let $\tilde{p}_{ig}(c)$ denote the fraction of people from cohort $c$ and group $g$ who choose occupation $i$, a choice made when they are young. Aggregating across people, the solution to the individual’s choice problem leads to

$$\tilde{p}_{ig}(c) = \frac{\bar{w}_{ig}(c)^{\theta}}{\sum_{s=1}^{M} \bar{w}_{sg}(c)^{\theta}} \quad \text{where} \quad \bar{w}_{igc} \equiv \frac{(T/3)w_{i}(c)s_{i}(c)\phi_{i}(c)[(1 - s_{i}(c))z_{ig}(c)]^{\frac{1-\eta}{3}}}{\bar{\tau}_{ig}(c,c)}. \quad (11)$$

\textsuperscript{11}Even though we model $T_{g}^{\text{home}}$ as a productivity term, it is isomorphic to a group specific preference for working in the home sector. Therefore, this term also captures changing social norms for women working in the market sector or changes in fertility and marriage patterns over time resulting in different incentives for groups to stay at home.

\textsuperscript{12}Proofs of the propositions are given in http://www.stanford.edu/ chadj/HHJKAppendix.pdf.
Remember $\tau_{ig}(c, c) \equiv \frac{(1+\tau^h_{ig}(c))^{\eta}}{1-\tau^h_{ig}(c)}$ is a composite of $\tau^h$ and $\tau^w$ facing cohort $c$ when young ($t = c$). Equation (11) says that occupational sorting depends on $\tilde{w}_{ig}$, which is the overall reward that someone from group $g$ with the mean talent obtains by working in occupation $i$, relative to the power mean of $\tilde{w}$ for the group over all occupations. The occupational distribution is driven by relative returns and not absolute returns: forces that only change $\tilde{w}$ for all occupations have no effect on the occupational distribution. Occupations where the return to skill $w_i$ is high relative to other occupations are ones where more people choose to work. However, all else equal, heterogeneity in $w_i$ will have the same effect on occupational choice of every group. The forces that potentially result in different occupational choices across groups are $z$, $\tau^w$, and $\tau^h$. The fraction of members of group $g$ that choose occupation $i$ is low when a group does not have a preference for that occupation ($z_{ig}$ is low), when employers discriminate against the group in that occupation ($\tau^w_{ig}$ is high), or when the group faces a friction in accumulating human capital in that occupation ($\tau^h_{ig}$ is high). It is useful to note that the three factors driving differential occupational choice across groups are isomorphic in Equation (11). However, we will show later that although these forces have the same effect on occupational choice, they leave different tell-tale signs in the data, which we will exploit in the empirical section of the paper.

2.4. Labor Force Participation

After the individual chooses a market occupation, she decides between exercising her skills in the chosen market occupation and working at home. Conditional on choosing market occupation $i$, her per period consumption from the home sector is given by Equation (9) where acquired human capital $h$ is the same as that in the market sector. Because the talent in the home sector $\epsilon_{\text{home}}$ is drawn from an extreme value distribution, indirect utility in the home sector can also be characterized as an extreme value distribution. The probability of choosing the market sector is thus the probability that draws from one extreme value distribution (utility in the chosen market sector) is greater than draws from another extreme value distribution (utility from the home sector). We aggregate the individual choices from this decision to obtain the share of individuals (conditional on choosing occupation $i$) that work in the market.

\footnote{See Luttmer (2008) for a similar result.}
Proposition 2 (Labor Force Participation): Let $LFP_{ig}(c, t)$ denote the fraction of people in cohort $c$ and group $g$ at time $t$ who chose occupation $i$ that decide to work rather than stay at home. This fraction is given by

$$LFP_{ig}(c, t) = \frac{1}{1 + \tilde{p}_{ig}(c) \cdot \left[ \frac{T_{g,home}(c, t)}{T_{ig}(c, t)} \cdot \frac{T_{ig}(c, t)}{1 - \tau_{ig}(c, t) - \tau_{ig}(c)} \cdot w_{i}(t) \right]}.$$

(12)

We do not observe $\tilde{p}$ or $LFP$ in the data, but their product is the fraction of people of a cohort-group actually working in an occupation, $p_{ig}$, which is the model’s counterpart of the occupational shares we do observe in the data:

$$p_{ig}(c, t) = \tilde{p}_{ig}(c) \cdot LFP_{ig}(c, t).$$

(13)

Remember $\tilde{p}_{ig}(c)$ is fixed for a given cohort because individuals choose their occupation when they are young. The model thus interprets changes in the occupational share of a given cohort over time as being driven entirely by changes in the labor force participation rate $LFP$. For a given cohort, $LFP$ increases when the wage for per unit of human capital $w_i$ rises, labor market discrimination $\tau_w$ declines, return to market talent $T_{ig}$ increases, and average talent in the home sector $T_{g,home}$ falls. For example, a decline in labor market discrimination facing female lawyers will increase the fraction of women of a given cohort working as lawyers. A decline in $w_i$ in low skilled occupations will lower the share of individuals (of a given cohort) working in the low skill occupations (for all groups).

2.5. Worker Quality

The sorting model then generates the average quality of workers in an occupation for each group. We show this in the following proposition:

Proposition 3 (Average Quality of Workers): For a given cohort $c$ of a group $g$ at time $t$, the average quality of workers in each occupation, including both human capital and talent, is

$$E [h_{ig}(c, t) \cdot e_{ig}(c, t)] = \gamma s_i(c) \phi_i(t) \left[ \left( \frac{\eta \cdot s_i(c) \phi_i(c) \cdot T_{ig}(c, c) \cdot w_i(c) \cdot (1 - \tau_{ig}(c))}{1 + \tau_{ig}(c)} \right)^\eta \left( \frac{1}{p_{ig}(c, t)} \right)^{\frac{1}{\eta}} \right].$$

(14)
where $\gamma \equiv \Gamma(1 - \frac{1}{\theta} \cdot \frac{1}{1-\eta})$ is related to the mean of the Fréchet distribution for abilities.

Notice that average quality is inversely related to the share of the group working in the occupation $p_{ig}(c, t)$. This captures the selection effect. For example, the model predicts that if the labor market discriminated against lawyers in 1960 only the most talented female lawyers would have chosen to work in this occupation in 1960. And as the barriers faced by female lawyers declined after 1960, less talented female lawyers moved into the legal profession and thus lowered the average quality of female lawyers. Conversely, in 1960, the average quality of white male lawyers would have been lower than it would have been if there were no discrimination against women and blacks.

### 2.6. Occupational Wages

Next, we compute the average wage for a given group working in a given occupation — the model counterpart to what we observe in the data.

**Proposition 4 (Occupational Wages):** Let $\overline{\text{wage}}_{ig}(c, t)$ denote the average earnings in occupation $i$ by cohort $c$ of age $a$ of group $g$. Its value satisfies

$$
\overline{\text{wage}}_{ig}(c, t) \equiv (1 - \tau_{ig}(t)) \cdot w_{i}(t) \cdot T_{ig}(c, t) \cdot E\left[h_{ig}(c, t) \cdot \epsilon_{ig}(c, t)\right] \\
= \gamma \bar{\eta} \left(\frac{m_{g}(c, t)}{LFP_{ig}(c, t)}\right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}} \cdot ((1 - s_{i}(c))z_{ig}(c))^{-1/\beta} \cdot \frac{(1 - \tau_{ig}(t))w_{i}(t)}{(1 - \tau_{ig}(c))w_{i}(c)} \cdot \frac{T_{ig}(c, t) \cdot s_{i}(c)\phi_{i}(c)}{s_{i}(c)\phi_{i}(c)}.
$$

(15)

where $m_{g}(c, t) = \sum_{i=1}^{M} \tilde{w}_{ig}(c, t)^{\theta}$. For the young cohort, individuals by definition, are in the first period of their lifecycle such that $t = c$ implying $\frac{T_{ig}(c, t)}{T_{ig}(c, c)} = 1$, $\frac{s_{i}(c)\phi_{i}(c)}{s_{i}(c)\phi_{i}(c)} = 1$, and $\frac{(1 - \tau_{ig}(t))w_{i}(t)}{(1 - \tau_{ig}(c))w_{i}(c)} = 1$. Thus, after controlling for the labor force participation rate, average earnings for a given group among the young differs across occupations only because of the term $[(1 - s_{i}(c))z_{ig}(c)]^{-1/\beta}$. Occupations in which schooling is especially productive (a high $\phi_{i}$ and therefore a high $s_{i}$) will have higher average earnings. Similarly, occupations where individuals have a strong disutility from being in the profession ($z_{ig}$ is small) have higher wages as compensation for the lower utility. And these are the only two forces that generate differences in wages across occupations for the young (controlling for labor force participation). Average earnings are no higher in occupations where a group faces less discrimination or lower frictions in human capital attainment.
or a better talent pool or a higher wage per efficiency unit. The reason is that each of these factors leads lower quality workers to enter those jobs. This composition effect exactly offsets the direct effect on earnings when the distribution of talent is Fréchet.

The exact offset due to selection is a feature of the Fréchet distribution, and we would not expect this feature to hold more generally. However, the general point is that when the selection effect is present, the wage gap is a poor measure of the frictions faced by a group in a given occupation. Such frictions lower the wage of the group in all occupations, not just in the occupation where the group encounters the friction. In the empirical section, we will examine the extent to which changes in the occupational distortions account for the narrowing of wage gaps.

Equation (15) also identifies the model the forces behind wage changes over cohorts life-cycle. For a given cohort-group in an occupation (say female lawyers born in the 1950s), \( s_i \) and \( z_{ig} \) are fixed. Therefore, the average wage increases over time when the price of skills in the occupation \( w_i \) increases, labor market discrimination \( \tau^w \) falls, return to experience is positive \( \frac{T_{ig}(c,t)}{T_{ig}(c,c)} > 1 \), the return to schooling increases \( \phi(t) > \phi(c) \), or the share of cohort-group in the occupation falls. Comparing wage changes across groups, the effect of the returns to schooling, experience, and returns to skill have the same effect on all groups (of a given cohort in the occupation). Thus, differences in the growth rate of wages between groups (say between men and women) can only be due to differences in the change in \( \tau^w \) between the groups (after controlling for the effect of changes in the share of the group in the occupation). We will use this insight to estimate the change in \( \tau^w \) in the empirical section.

Putting together the equations for the occupational shares in (11) and (13), labor force participation in (12), and wages in each occupation in (15), we get the relative propensity of a group to work in an occupation for the young cohort in each year

**Proposition 5 (Relative Propensities):** Focusing on the young cohort in each year, the fraction of a group working in an occupation — relative to white men — is given by

\[
\frac{p_{ig}(c,c)}{p_{i,wm}(c,c)} = \left( \frac{T_{ig}(c,c)}{T_{i,wm}(c,c)} \right)^\theta \left( \frac{\tau_{ig}(c,c)}{\tau_{i,wm}(c,c)} \right)^{-\theta} \left( \frac{\text{wage}_{ig}(c,c)}{\text{wage}_{i,wm}(c,c)} \right)^{-\theta(1-\eta)}
\]

Equation (16) is one of our key theoretical relationships that will we will take to the data. The proposition states that the propensity of a group to work in an occupation
(relative to white men) depends on three terms: relative mean talent in the occupation (which we assume to be one in our base specification), the relative composite occupational frictions, and the average occupational wage gap between the groups. From Proposition 2, the wage gap itself is a function of the distortions faced by the group and the price of skills in all occupations. With data on occupational shares and wages, we can measure a composite of relative mean talent and occupational frictions. The preference parameters $z_{ig}$ do not enter this equation once we have controlled for the wage gap; instead, they influence the wage gaps themselves. We will use these insights to help us recover both the composite frictions and preference parameters.

### 2.7. Equilibrium

A competitive equilibrium in this economy consists of individual choices \( \{C, e, s\} \), an occupational choice in the first period, a labor force participation decision in each period, total efficiency units of labor of each group in each occupation \( H_{ig} \), final output \( Y \), and an efficiency wage \( w \) in each occupation such that

1. Given an occupational choice, the occupational wage \( w \), and idiosyncratic ability \( \epsilon \) in that occupation, each individual chooses \( c, e, s \) to maximize utility from market work in the first period:

\[
U(\tau_w, \tau_h, w, \epsilon) = \max_{c, e, s} (1 - s) \cdot c^{\beta} \cdot z_{ig} \text{ s.t. } c = (1 - \tau_{ig}^w) \cdot w \cdot \epsilon \cdot T \cdot h - e \cdot (1 + \tau_{ig}^h).
\]

(17)

2. Each individual chooses the occupation that maximizes his or her utility from market work in the first period:

\[
i^* = \arg \max_{i} U(\tau_{ig}^w, \tau_{ig}^h, z_{ig}, w_i, \epsilon_i),\text{ taking as given } \{\tau_{ig}^w, \tau_{ig}^h, z_{ig}, w_i, \epsilon_i\}.
\]

3. Each individual chooses between the market occupation and the home sector in each period taking the choice of market occupation, human capital \( h, w, \tau_w, \tau_h, z \), idiosyncratic ability in the market \( \epsilon \) and in the home sector \( \epsilon_{\text{home}} \) as given.

4. A representative firm in the final good sector hires \( \sum_g H_{ig} \) in each occupation to maximize profits net of utility cost of discrimination given by equation (2).

5. A representative firm in the education sector maximizes profit net of the utility cost of discrimination given by equation (3).
The equations characterizing the general equilibrium are given in the next result.

**Proposition 6** (Solving the General Equilibrium): *The general equilibrium of the model is \( \{ H_{ig}^{\text{supply}}, H_i^{\text{demand}}, w_i \} \) and \( Y \) such that*

1. \( H_{ig}^{\text{supply}}(t) \) aggregates the individual choices:
   \[
   H_{ig}^{\text{supply}}(t) = \sum_c q_g(c)p_{ig}(c,t)T_{ig}(c,t) \cdot \mathbb{E}[h_{ig}\epsilon_{ig}(c,t) \mid \text{Person chooses } i] \tag{18}
   \]
   and the average quality of workers is given in equation (14).

2. \( H_i^{\text{demand}}(t) \) satisfies firm profit maximization:
   \[
   H_i^{\text{demand}}(t) = \left( \frac{A_i(t)^{\frac{\sigma-1}{\sigma}}}{w_i(t)} \right)^\sigma Y(t) \tag{19}
   \]

3. \( w_i(t) \) clears each occupational labor market:
   \[
   \sum_g H_{ig}^{\text{supply}}(t) = H_i^{\text{demand}}(t).
   \]

4. Total output \( Y(t) \) is given by the production function in equation (1).

2.8. Intuition

To develop intuition, consider the following simplified version of the model. First, assume only two groups, white men and white women, and that white men face no distortions. Second, assume occupations are perfect substitutes \( (\sigma \rightarrow \infty) \) so that \( w_i = A_i \). With this assumption, the production technology parameter pins down the wage per unit of human capital in each occupation. In addition, \( \tau_{i,ww} \) affects the average wage and occupational choices of white women \( (g = ww) \) but has no effect on white men. Third, assume \( \phi_i = 0 \) (no schooling time), \( T_{ig} = 1 \forall g \) (mean occupational talent is the same for every group), and \( z_{ig} = 0 \). Finally, assume that each cohort lives for one period and all workers choose to work in the market.

The average wages of white men and white women, respectively, are then given by:

\[
\text{wage}_{wm} = \left( \sum_{i=1}^N A_i^g \right)^{\frac{1}{\sigma}} \frac{1}{1-\eta} \tag{20}
\]
The average white male wage is a power mean of the occupational productivity terms and is not affected by the occupational distortions facing white women (this is driven by the assumption that occupations are perfect substitutes). The average wage of white women is a power mean of the occupational productivities and distortions.

Aggregate consumption and output can then be expressed as a function of the average wage of white men and women. Aggregate consumption of workers is proportional to the aggregate wage. Aggregate output is the sum of consumption, investment in human capital, and profits of the firm in the final goods sector and the producer of good used for human capital:

\[
Y = (1 - \eta) \cdot \left( q_m \cdot \bar{wage}_m + q_w \cdot \bar{wage}_w \right) \\
+ \eta \cdot \left( q_m \cdot \bar{wage}_m + q_w \cdot \frac{\bar{wage}_w}{1 + \tau^h} \right) \\
+ q_w \cdot \bar{wage}_w \cdot \left( \frac{\tau^w}{1 - \tau^w} + \eta \cdot \frac{\tau^h}{1 + \tau^h} \right)
\]  

(22)

where \( q_w \) and \( q_m \) denote the number of white women and men and \( \bar{w}^w \) and \( \bar{w}^h \) denote the earnings-weighted average of the labor market friction and human capital friction facing women.\(^{14}\) The first row denotes aggregate worker consumption. The second row denotes aggregate investment in human capital. The third row denotes profits of the final goods firm and in the human capital sector. Profits of the final goods firm are increasing in \( \bar{\tau}^w \) and compensate the owner for her disutility from employing workers she dislikes. Profits in the human capital sector are increasing in \( \bar{\tau}^h \) and compensate providers of human capital to compensate them for the disutility of providing human capital to women.

To see the effect of the distortions on aggregate consumption and output, assume \( \tau^h = 0 \) and \( 1 - \tau^w_i \) and \( A_i \) are jointly log-normally distributed. The average female wage

\[^{14}\bar{\tau}^w = \sum_{i=1}^{M} \omega_i \tau^w_i \]  

where \( \omega_i \equiv \frac{\frac{p_w}{\bar{w}^w_i}}{\sum_{j=1}^{M} \frac{p_w}{1 - \tau^w_j}} \). \( \bar{\tau}^h \) is defined in a similar manner.
The allocation of talent

\[
\ln \text{wage}_{w} = \ln \left( \sum_{i=1}^{N} A_i^\theta \right)^{\frac{1}{\theta}} + \frac{1}{\eta - 1} \ln (1 - \tilde{\tau}^w) + \frac{\eta}{1 - \eta} \cdot \ln (1 - \tilde{\tau}^w) - \frac{1}{2} \cdot \frac{\theta - 1}{1 - \eta} \cdot \text{Var}(\ln(1 - \tau^w)).
\]

(23)

The first term says that the average female wage is increasing in the power mean of occupational productivities. The second and third terms state that the average female wage is decreasing in the weighted average of the labor market frictions, and more so the higher is \( \eta \) (the greater the importance of goods for human capital). The fourth term says that the average female wage is decreasing in the dispersion of \( 1 - \tau^w \).

Both the mean and dispersion of \( \tau^w \) lower the average female wage and aggregate consumption. However, the effect of the mean of \( \tau^w \) on aggregate output is different from the effect of the dispersion of \( \tau^w \). The mean of \( \tau^w \) has two effects on the average wage. First, \( \bar{\tau}^w \) has direct effect on the average wage: The elasticity of the average wage to \( 1 - \bar{\tau}^w \) is one. This effect is given by the second term in equation (23). This effect is simply a redistribution from firms: workers get a larger share and firm owners a smaller share of aggregate output, with no change in aggregate output.

The mean of \( \tau^w \) also has an indirect effect on the average wage by changing the return to investment in human capital. The magnitude of this effect depends on \( \eta \), and is captured by the third term in (23). The net effect of a change in \( \bar{\tau}^w \) on the aggregate wage is larger than aggregate output, where the gap depends on \( \eta \). When \( \eta = 0 \) then \( \bar{\tau}^w \) only has a distributional effect: whatever gain (loss) in wages is entirely offset by a corresponding loss (gain) in profits, and aggregate output is unchanged.

The dispersion of \( \tau^w \) across occupations affects the wage via a different channel. Here, dispersion of \( \tau^w \) affects the allocation of labor across occupations. A decline in the dispersion of \( \tau^w \) improves the allocation of labor, which increases the average wage, but has no effect on the split of aggregate output between wages and profits. The dispersion of \( \tau^w \) has the same effect on the aggregate wage as on aggregate output.

This logic also applies to the effect of the mean vs. the dispersion of \( \tau^h \). The mean value of \( \tau^h \) has a larger effect on wages than on output because part of the wage gain is a redistribution from human capital providers to workers in the form of additional investment in human capital. In contrast, the dispersion of \( \tau^h \) has no effect on the distribution between workers' investment in human capital and profits of human capital.
providers. Therefore, the dispersion of \( \tau^w \) has the same effect on the aggregate wage as on aggregate output.

Though we will not impose these simplifying assumptions, with this motivation in mind we will later isolate the effect of change in the dispersion vs. the mean of the frictions on aggregate consumption and aggregate output. This simple example provides the intuition for these later results.

3. Data

We use data from the 1960, 1970, 1980, 1990, and 2000 Decennial Censuses and the 2010-2012 American Community Surveys (ACS).\(^{15}\) We make four restrictions to the data when performing our analysis. First, we restrict the sample to white men (wm), white women (ww), black men (bm) and black women (bw). These will be the four groups we analyze in the paper. Second, we only include individuals between the ages of 25 and 55. This restriction focuses the analysis on individuals after they finish schooling and prior to retirement. Third, we exclude individuals on active military duty. Finally, we exclude individuals who report being unemployed (not working but searching for work). Our model is not well suited to capture transitory movements into and out of employment. Appendix Table B1 reports summary statistics from our sample.\(^{16}\)

We do not have panel data. Instead, we follow synthetic cohorts over time. We define three age periods within a cohort’s lifecycle: the young (those aged 25-34), the middle aged (those aged 35-44) and the old (those aged 45-55). A given synthetic cohort, for example, would be the young in 1960, the middle aged in 1970, or the old in 1980. We have information on 8 cohorts for the time periods we study. For 4 cohorts, we observe information at all three life cycle points. We observe either one or two life cycle points for the remaining cohorts.

A key to our analysis is a consistent definition of occupations over time. First, we treat the home sector as a distinct occupation. We define a person who is not currently employed or who works less than ten hours per week as being in the home sector.

\(^{15}\)When using the 2010-2012 ACS data, we pool all three years together and treat them as one cross section. Henceforth, we refer to the pooled 2010-2012 sample as the 2010 sample.

\(^{16}\)For all analysis in the paper, we apply the sample weights available in the different surveys.
Those who are employed but usually work between ten and thirty hours per week are classified as part-time workers. We split the sampling weight of part-time workers equally between the home sector and the occupation in which they are working. Individuals working more than thirty hours per week are considered to be full-time in an occupation outside of the home sector. Second, we define the non-home occupations using the roughly 67 occupational sub-headings from the 1990 Census occupational classification system. Appendix Table B2 reports the 67 occupations we analyze. Some samples of the occupational categories are “Executives, Administrators, and Managers”, “Engineers”, “Natural Scientists”, “Health Diagnostics”, “Health Assessment”, and “Lawyers and Judges”. In prior versions, we also used a more detailed classification of occupations by using 340 three digit occupation groupings that were defined consistently since 1980, as well as aggregating occupations into 20 broad occupational groups defined consistently since 1960. Our results were broadly similar at these different levels of occupation aggregation. We choose the 67 occupations because these give us the largest amount of consistently classified occupations during the 1960-2010 period.

We measure earnings as the sum of labor, business, and farm income in the previous year. For earnings we restrict the sample to individuals who worked at least 48 weeks during the prior year, who earned at least 1000 dollars (in 2007 dollars) in the previous year, and who reported working more than 30 hours per week. We define the hourly wage as total annual earnings divided by total hours worked in the previous year. We convert all earnings data from the Census to constant dollars.

4. Data Inference

As discussed earlier, $\tau^h$, $\tau^w$, and $z$ are isomorphic in terms of their effect on occupational choice. We will exploit two features of the model to discriminate between these forces. First, whereas occupation specific labor market discrimination and human capital frictions do not affect wages in the corresponding occupation (they lower overall wages for the discriminated group), preferences to work in an occupation do affect a

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17 See http://usa.ipums.org/usa/volii/99occup.shtml. We use the 1990 occupation codes as our basis because the 1990 codes are available in all Census and ACS years since 1960. We start our analysis in 1960, as this is the earliest year for which the 1990 occupational crosswalk is available.

18 Appendix Table B3 gives a more detailed description of some of these occupational categories.
group’s average wage in that occupation. This will allow us to distinguish the effect of a composite of $\tau^w$ and $\tau^h$ from the effect of preferences $z$. Second, we will assume that labor market discrimination affects all working cohorts of a discriminated group equally in a given year. In contrast, human capital frictions for a given group in a given year only affects the young cohort of the group in that year. This distinction will allow us to separately identify $\tau^w$ and $\tau^h$.

4.1. Labor Market and Human Capital Frictions vs. Occupational Preferences

To begin, we rearrange equation (16) to solve for a composite measure of distortions in the labor and human capital markets:

$$\hat{\tau}^i(c, c) \equiv \frac{\tau^i(c, c)}{\tau^i_{wm}(c, c)} = \frac{T^i(c, c)}{T^i_{wm}(c, c)} \left( \frac{p^i(c, c)}{p^i_{wm}(c, c)} \right)^{1/\theta} \left( \frac{\text{wage}^i(c, c)}{\text{wage}^i_{wm}(c, c)} \right)^{(1-\eta)}$$

(24)

Suppose we normalize $\tau^h$ and $\tau^w$ of white men to zero and $\frac{T^i}{T^i_{wm}}$ to one.\textsuperscript{19} Equation (24) then says that we can recover the composite $\tau^i(c, c)$ from three pieces of data: the share of the group working in the occupation (out of all working individuals) relative to that of white men, the average wage gap of a group relative to white men in an occupation, and estimates of $\theta$ and $1-\eta$. Intuitively, when the share of the group in the occupation is low (after we control for the wage gap), we infer the group is discouraged from entering the occupation because they face discrimination in the labor market or frictions in acquiring the human capital necessary for the occupation.

We now present the ingredients needed to measure $\tau^i$ from equation (24). Figure 1 plots the dispersion in occupations of young white women, black men and black women (relative to white men) over time. Specifically, the figure presents the standard deviation of $\ln(p^i/p^i_{wm})$ across occupations for the young cohort in each decade.\textsuperscript{20} As shown in Figure 1, the occupations of white men and white women have converged over time. In particular, the standard deviation of $\ln(p^i/p^i_{wm})$ fell sharply from 1960

\textsuperscript{19}This will be our base normalization throughout the paper. However, in our robustness specifications, we explore a variety of other normalizations including the assumption that the $\tau$’s for white women are zero and that men have an absolute advantage in brawny occupations relative to women. The latter will imply that the relative $T$’s between men and women will not equal one in some occupations.

\textsuperscript{20}We compute the standard deviation of $(\ln(p^i) - \ln(p^i_{wm}))$ across occupations weighting each occupation by the share of earnings in that occupation. We exclude the home sector in this calculation.
Figure 1: Standard Deviation of Relative Occupational Shares

Note: Figure shows the time series evolution of the standard deviation of the occupational propensities for young white women relative to young white men (solid fill) and young black men relative to young white men (hatch fill). Specifically, for each year, we compute the standard deviation of $\ln(p_{i,ww}/p_{i,wm})$ and $\ln(p_{i,bm}/p_{i,wm})$ for young individuals across occupations. When computing the standard deviation in the ratio across occupations, we weight each occupation by its share of earnings out of total earnings.

through 2000. Between 2000 and 2010, the standard deviation in occupational propensities has remained constant. For black men, the standard deviation of $\ln(p_{ig}/p_{i,wm})$ also fell sharply between 1960 and 1980 and has remained relatively constant since. When filtered through equation (24), the decline in the dispersion of $\ln(p_{ig}/p_{i,wm})$ implies that the dispersion of the combination of $\tau^w$, $\tau^h$ and $z$ has declined over time.

The differences in occupational propensities shown in Figure 1 within a given year can be driven by differential dispersion in labor market market discrimination, human capital frictions, or occupational preferences. Likewise, the convergence in occupational propensities since 1960 can be driven by a decline in the dispersion in any of these three forces. However, a smaller dispersion in occupational preferences will show up as smaller dispersion in wage gaps across occupations, whereas convergence in the labor market discrimination and human capital frictions will not affect the dispersion in wage gaps across occupations given that these factors should affect the wage gap equally in all occupations. Specifically, the wage gap in occupation $i$ for group $g$ vs.
white men of the same cohort is:

\[
\frac{\text{wage}_{ig}(c, c)}{\text{wage}_{i,wm}(c, c)} = \left( \frac{m_{ig}(c, c)}{m_{i,wm}(c, c)} \cdot \frac{LFP_{i,wm}(c, c)}{LFP_{ig}(c, c)} \right)^{\frac{1}{(1-\eta)}} \cdot \left( \frac{z_{ig}(c)}{z_{i,wm}(c)} \right)^{-1/\beta} \tag{25}
\]

The wage gap in an occupation thus isolates the effect of \(z_{ig}\) on the occupational choice patterns in Figure 1 (conditional on labor force participation). Thus, when we condition the occupational gaps on the wage gap in the occupation in (24), we isolate the effect of the labor market and human capital distortions on occupational choice. With this logic in mind, Figure 2 plots the standard deviation of the wage gap across market occupations for young (25-34 year old) white women, black men, and black women relative to white men in each year. Occupations are weighted by their share of earnings. As can be seen, the standard deviation of the wage gap across occupations has fallen sharply for each group (relative to white men) between 1960 and 2010, with much of this decline occurring prior to 1990. The decline in the standard deviation of the wage gap suggests that the dispersion of \(z_{ig}\) has declined over time, and this decline is likely to be partially responsible for narrowing of the gap in the occupational distribution.

4.2. Estimating \(\theta\) and \(\eta\)

Wages within an occupation for a given group should follow a Fréchet distribution with the shape parameter \(\theta(1 - \eta)\). This reflects both comparative advantage (governed by \(1/\theta\)) and amplification from endogenous human capital accumulation (governed by \(1/(1 - \eta)\)). Using micro data from the U.S. Population Census/ACS described earlier, we estimate \(\theta(1 - \eta)\) to fit the distribution of the residuals from a cross-sectional regression of log hourly wages on 66x4x3 occupation-group-age dummies in each year. We use MLE, with the likelihood function taking into account the number of observations which are top-coded in each year. The resulting estimates for \(\theta(1 - \eta)\) range from a low of 1.24 in 1980 to a high of 1.42 in 2000, and average 1.36 across years.\(^{21}\)

The parameter \(\eta\) denotes the elasticity of human capital with respect to education spending and is equal to the fraction of output spent on human capital accumulation. Spending on education (public plus private) as a share of GDP in the U.S. averaged 6.6

\(^{21}\)Sampling error is minimal because there are 300-400k observations per year for 1960 and 1970 and 2-3 million observations per year from 1980 onward. We did not use 2010 data because top-coded wage thresholds differed by state in that year.
Figure 2: Standard Deviation of Wage Gaps by Decade

Note: Figure shows the time series evolution of the standard deviation of the wage gaps for young white women relative to young white men (solid fill) and young black men relative to young white men (hatch fill). Specifically, for each year, we compute the standard deviation of the log occupational wage gaps across occupations. As suggested by the model, we focus on the wage gaps of the young (those aged 25-34). When computing the cross occupation standard deviation in the wage gaps, we weight each occupation by its share of earnings out of total earnings.
percent over the years 1995, 2000, 2005, and 2010. Since the labor share in the U.S. in the same four years was 0.641 and the average share of the young relative to all workers was 0.286, this implies an \( \eta \) of .358. With our base estimate of \( \theta(1 - \eta) = 1.36 \), \( \eta = 0.358 \) gives us \( \theta = 2.12 \).

An alternative way to estimate \( \theta \) is to use information on the elasticity of labor supply. In our model, the extensive margin elasticity of labor supply with respect to a wage change is equal to \( \theta \cdot \frac{1-LFP_g(c,a)}{LFP_g(c,a)} \), where \( LFP_g(c,a) \) is the labor force participation rate of group \( g \) of cohort \( c \) at age \( a \). With data on labor force participation rates for different cohort-groups, we can back out \( \theta \) from estimates of the labor supply elasticity. The meta analysis in Chetty et al. (2012) suggests an extensive margin labor supply elasticity of about 0.26. The underlying data in their meta analysis come from the 1970-2007 period. In 1990, roughly in the middle of their analysis, 89.9 percent of men aged 25-34 were in the labor force. To match a labor supply elasticity of 0.26, our model implies that \( \theta \) would equal 2.31, which is a little higher than the estimate of \( \theta \) we get from the wage dispersion. We will use \( \theta = 2.12 \) as our base case, but will also provide estimates for \( \theta \) of 4, which is on the upper end of the estimates for labor supply elasticities.

### 4.3. Estimating Composite \( \tau \)'s

We now present the estimates of \( \tau_{ig} \) computed from data on the employment shares and relative wages in each occupation (and assuming that \( \theta = 2.12 \) and \( \eta = 0.358 \)). Figure 3 displays \( \tau_{ig} \) for white women for a select subset of occupations. As shown, \( \tau_{ig} \) was very high for women in 1960 in the construction, lawyer, and doctor occupations relative to the teacher and secretary occupations. \( \tau_{ig} \) for white women lawyers and doctors in 1960 hovered around 4. If \( \tau_{ig} \) is only driven by labor market frictions, it is as if women lawyers in 1960 received as wages only a fourth of their marginal products relative to their male counterparts. The model infers large \( \tau_{ig} \)'s for white women in these occupations in 1960 because there were few white women doctors and lawyers in 1960, even after controlling for the gap in wages between women lawyers and male lawyers. Conversely, a white woman in 1960 was 24 times more likely to work as a

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23. Labor share data are from https://research.stlouisfed.org/fred2/series/LABSHPUSA156NRUG. The young's share of earnings is from the U.S. Population Census/ACS.
secretary than was a white man. The model explains this enormous gap by assigning a $\tau_{ig}$ of about 0.8 for white women secretaries. When $\tau_{ig}$ is below 1, it is as if women are subsidized in the sector. Although we will interpret this as indicating that white women were subsidized in 1960 to become secretaries, it can also be the case that males who chose to be secretaries were discriminated against or that women simply had an absolute advantage as secretaries. We will later consider the effect of these alternative normalizations.

Over time, white women saw large declines in $\tau_{ig}$ for the lawyer and doctor occupations, while there was a more modest decline in their $\tau_{ig}$ for construction. As of 2010, white women faced only small composite frictions in the lawyer, doctor, and teacher occupations. However, the composite friction for white women in the construction sector remained large. As with secretaries, the construction patterns could be the result of women having a comparative disadvantage (relative to men) as construction workers.

Figure 4 shows the estimated $\tau_{ig}$ for the same set of occupations for black men and black women respectively. A similar overall pattern emerges, with the $\tau_{ig}$ being substantially above 1 in general in 1960, but falling through 2010. Still, they remained above 1 by 2010, especially for the high-skilled occupations, suggesting barriers remain. As for white women, the largest decline occurred in high-skilled occupations like doctors and
Figure 4: Estimated Barriers ($\hat{\tau}_{ig}$) for Black Men and Black Women

Note: Author’s calculations based on equation (24) using Census data and imposing $\theta = 2.12$ and $\eta = 0.358$. The results for Black Men are shown in the left panel while the results for Black Women are shown in the right panel.

Figure 5: Mean and Variance of Composite Barrier ($\hat{\tau}_{ig}$) by Group

Note: The left panel shows the average level of $\hat{\tau}_{ig}(c,c)$, weighted by average earnings in each occupation. The right panel shows the variance of the log of that measure, weighted in the same way.

lawyers. Unlike for white women, almost the entire change in the $\tau_{ig}$ for black men occurred prior to 1980. The plots for black women look like a combination of those for white women and black men.

The left panel of Figure 5 shows the weighted average of $\tau_{ig}$ for white women, black men, and black women, respectively. For white women, the mean $\tau_{ig}$ fell from about 3.5 in 1960 to approximately 1.5 in 2010. The decline was linear through 1990 and then slowed thereafter. Black men experienced a decline in mean $\tau_{ig}$ from about 2 in 1960 to 1.5 in 2010. Essentially all the decline occurred by 1980.

The right panel of Figure 5 shows the weighted standard deviation of $\tau_{ig}$ across occupations for white women, black men and black women. In our model, it the dis-

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$^{24}$ The weights are the occupation’s share of earnings out of total earnings. In each survey year, we update the weights to reflect how the occupation share of earnings out of total earnings evolves over time.
persion of \( \tau_{ig} \) that generates misallocation of talent across occupations. As seen from the figure, all three groups experienced a decline in the standard deviation of \( \tau_{ig} \) across occupations between 1960 and 2010.

### 4.4. Labor Market vs. Human Capital Frictions

The estimates of \( \tau_{ig} \) shown above are a combination of the labor market and human capital market frictions. We cannot separately identify the level of \( \tau_{ig}^w \) from that of \( \tau_{ig}^h \) but we can distinguish the changes in \( \tau_{ig}^w \) versus \( \tau_{ig}^h \) exploiting lifecycle variation. The key assumption that allows us to do this is that \( \tau_{ig}^w \) affects all cohorts in the labor market at the same point in time, whereas \( \tau_{ig}^h \) only affects the young. We utilize two pieces of data to do this.

First, the wage gap of cohort \( c \) and group \( g \) (relative to white men) in occupation \( i \) at time \( t \) relative to the wage gap at time \( c \) (when cohort \( c \) was young) is

\[
\frac{\text{gap}_{ig}(c,t)}{\text{gap}_{ig}(c,c)} \propto \left( \frac{p_{i,wm}(c,t)/p_{i,wm}(c,c)}{p_{ig}(c,t)/p_{ig}(c,c)} \right)^{1/\theta} \cdot \frac{T_{ig}(c,t)/T_{ig}(c,c)}{T_{i,wm}(c,t)/T_{i,wm}(c,c)} \cdot \frac{1-\tau_{ig}^w(t)}{1-\tau_{ig}^w(c)}
\]

(26)

Equation (27) indicates that, ceteris paribus, a decline in \( \tau_{ig}^w \) increases the share of individuals of a given cohort and group that chose to work in the market occupation.

For a given cohort, differences across occupations in the change in the wage gap are a function of three things: (1) the change in occupational shares relative to white men; (2) the returns to experience of the group relative to white men; and (3) the change in \( \tau_{ig}^w \) over time. Therefore, we can back out the change in \( \tau_{ig}^w \) from data on the change in the wage gap for a given cohort in a given occupation, after controlling for the effect of changes in the occupational shares and the returns to experience. Intuitively, if labor market discrimination diminishes over time, this will raise the average wage in occupations where the group previously faced discrimination.

An alternative to using data on wages is to use information on the change in occupational shares of a given cohort over time. Specifically, equation (12) can be expressed as the ratio of the share of a cohort-group in an occupation at time \( t \) relative to time \( c \):

\[
\frac{p_{ig}(t)}{p_{ig}(c)} = \frac{1 - p_{ig}(t)}{1 - p_{ig}(c)} \cdot \left[ \frac{T_{i,wm}(c,t)/T_{ig}(c,t)}{(1-\tau_{ig}^w(c,t)) \cdot w(t)} \right]^{\theta} \cdot \left[ \frac{T_{i,wm}(c,c)/T_{ig}(c,c)}{(1-\tau_{ig}^w(c,c)) \cdot w(c)} \right]^{\theta}
\]

(27)
i. Since we can easily measure the change in the share of individuals of a given cohort that work in each market sector, we can use this information to back out the implied change in $\tau_{ig}^w$.

In practice, we pick the value of the change in $\tau_{ig}^w$ that provides the best fit to the change in the wage gap and occupational shares (but fits neither of the two data moments perfectly). Specifically, we assume the return to experience $\frac{\bar{w}_{ig}(c,t)}{\bar{w}_{ig}(c,c)}$ is given by the return to experience for white men (of the same cohort in the same occupation) adjusted by the change in the share of the cohort-group in the occupation.\(^{25}\) Note, this assumption allows women and men to have different observed lifecycle wage profiles in part because older women may have had less labor market experience than older men. What we assume is conditional on experience women and men would otherwise have the same lifecycle profile of wages. We then pick the combination of the change in $T_{home}^i$ and $\tau_{ig}^w$ to fit both the change in the wage gap and occupational shares in each occupation while exactly fitting the change in the aggregate labor force participation rate of a given cohort (across all occupations).\(^{26}\) For each cohort group, we have $2M$ moments ($M$ wage changes and $M$ changes in occupation shares for each of the $M$ market occupations) to estimate $M + 1$ parameters ($M$ estimates of the change in $\tau_{ig}^w$ and one estimate of the change in $T_{home}^i$).

The last step is to back out the change in $\tau_{ig}^h$ as the residual of the change in $\tau_{ig}$ after controlling for the change in $\tau_{ig}^w$.\(^{27}\) Again, the key identifying assumption is that labor market discrimination equally affects all cohorts of the discriminated group in the labor market at the same point in time, whereas discrimination in schooling only affect individuals in the human capital accumulation stage of their life-cycle.

---

\(^{25}\)We measure the returns to experience in occupation $i$ for white men in cohort $c$ as the change in the average wage in the occupation among the cohort after adjusting for the change in the wage per unit of human capital in the occupation $w_i(t)/w_i(c)$. Using the labor force participation rate of group $g$ and cohort $c$ in an occupation, we calculate the returns to experience as the product of $\frac{P_{ig}(c,c)}{P_{ig}(c,t)}$ and the returns to experience for white men of the same cohort in the same occupation. When the occupational share in the group declines, we assume the returns to experience for the group is equal to the returns of white men (of the same cohort in the same occupation).

\(^{26}\)The estimation takes as given the values of $w_i$ and $w_{home}$. We discuss later how we infer their values.

\(^{27}\)Specifically, we use $\frac{1+\tau_{ig}^h(t)}{1+\tau_{ig}(c)} = \left( \frac{\tau_{ig}(t)}{\tau_{ig}(c)} \cdot \frac{1-\tau_{ig}^w(t)}{1-\tau_{ig}^w(c)} \right)^{1/\eta}$.
4.5. Other Parameter Values and Exogenous Variables

We now discuss how we obtain the values of the remaining parameters and variables we need for the estimation. The parameters we assume constant over time are \( \eta, \theta, \sigma, \) and \( \beta. \) We discuss our estimates of \( \eta \) and \( \theta \) above. The parameter \( \sigma \) governs the elasticity of substitution among our 67 occupations in aggregating up to final output. We have little information on this parameter and choose an elasticity of substitution \( \sigma = 3 \) for our baseline value but explore the robustness of our results to other values. The parameter \( \beta \) is the geometric weight on consumption relative to time in an individual's utility function (4). As schooling trades off time for consumption, wages must increase more steeply with schooling when people value time more (i.e. when \( \beta \) is lower). We choose \( \beta = 0.693 \) to match the Mincerian return to schooling across occupations, which averages 12.7% across the six decades.\(^{28}\) Our main results on the importance of declining frictions to occupational choice for women and blacks is essentially invariant to our choice of \( \beta. \)

The remaining variables we need are \( \phi_i, w_i, w^{home} \) and \( A_i. \) The variable \( \phi_i \) governs the occupation-specific return to time invested in human capital. In the model, higher \( \phi_i \) raises time spent in (say) schooling and average wages in occupation \( i. \) We therefore infer \( \phi_i \) from data on average wages in each occupation among young white men in each year.\(^{29}\) To estimate \( w_i \) and \( w^{home}, \) we use the fact that \( w_i, \phi_i, \) and \( w^{home} \) collectively determine the observed share of young white men in each occupation and the average wage of young white men across all occupations.\(^{30}\) Using the estimates of \( \phi_i \) obtained from the data on wage gaps, we pick \( w_i \) and \( w^{home} \) to exactly fit the observed occupational shares and the average wage for young white men in each year. The intuition is that, conditional on estimates of \( \phi_i, \) the average wage for young white men pins down a weighted average of \( w_i \). The differences in occupational shares then pin down the heterogeneity in \( w_i \) across occupations: occupations with a large share of young

\(^{28}\)The average wage of group \( g \) in occupation \( i \) is proportional to \( (1 - s_i)^{\frac{\beta}{\psi}}. \) If we take a log linear approximation around average schooling \( \bar{s}, \) then \( \beta \) is inversely related to the Mincerian return to schooling across occupations (call this return \( \psi): \beta = (\psi(1 - \bar{s}))^{-1}. \) We calculate \( s \) as average years of schooling divided by a pre-work time endowment of 25 years, and find the Mincerian return across occupations \( \psi \) from a regression of log average wages on average schooling across occupation-groups, with group dummies as controls. We then set \( \beta = 0.693, \) the simple average of the implied \( \beta \) values across years. This method allows the model to approximate the Mincerian return to schooling across occupations.

\(^{29}\)We have \( M - 1 \) wage gaps in each year so we normalize \( \phi_i \) in one occupation. In each year We set \( \phi_i \) for farm non-managers to fit their average years of schooling relative to a 25 year time endowment.

\(^{30}\)This can be seen by combining equations (11), (12), and (13).
white men are ones where the price of skills $w_i$ is high. Conditional on $w_i$, the price of home talent $w^{home}$ pins down the aggregate labor force participation rate (which is simply the sum of the observed occupational shares for the market occupations). Lastly, knowing the $w_i$'s and the production function (1) allows us to back out the $A_i$'s.


We are now ready to present the evolution of the three key variables that generate occupational misallocation and wage gaps across groups. As discussed, we have a strategy for measuring the changes in $\tau_{i1g}$ and $\tau_{i2g}$. We normalize the initial level of $\tau_{i1g}$ and $\tau_{i2g}$ by assuming that 17 percent of the variation of $\tau_{ig}$ in 1960 was driven by variation in $\tau_{i1g}$ and the remainder due to variation in $\tau_{i2g}$. This normalization yields the same contribution of $\tau_{i1g}$ vs. $\tau_{i2g}$ to $\tau_{ig}$ variation in 1970–2010 as in 1960. In the subsequent section, we will explore robustness to alternative initial splits.

Figure 6 presents the weighted average (left panel) and weighted dispersion (right panel) of $\tau_{i1g}$, $\tau_{i2g}$, and $z_{ig}$ for white women. The decline in average $\tau$ shown in Figure 3 is almost entirely driven by the decline in $\tau_{i2g}$, with only a small decline in $\tau_{i1g}$ from 1960 to 1970. Falling dispersion in $\tau$ for white women was also largely due to declining dispersion of $\tau_{i2g}$ from 1960 to 1980. Over the same time period, labor market frictions $\tau_{i1g}$ and occupational preferences $z$ converged across occupations. These results suggest that it is primarily frictions in human capital attainment that kept women away from higher skilled occupations in the 1960s, 1970s, and 1980s, as opposed to discrimination in the labor market. We want to stress, however, that we also find a modest role for changing preferences in explaining some of the changing occupational choices of white women (relative to white men) over time.

Figure 7 presents the same variables for black men. The key difference relative to white women is the larger role of labor market discrimination in the case of black men. There is a sharp decline in both the mean and the dispersion of $\tau_{i1g}$ for black men, largely between 1960 and 1980. After 1980, there appears to be little change. Our results suggest that both barriers to human capital attainment and labor market discrimination will be important in explaining the changing occupational choice of black men (relative to white men) over time. Finally, Figure 8 presents the path of
Figure 6: Means and Variances of Frictions for White Women

Note: The left panel shows the average level of the frictions, weighted by total earnings in each occupation in 2010. The right panel shows the variance of the log frictions, weighted in the same way.

Figure 7: Means and Variances of Frictions for Black Men

Note: The left panel shows the average level of the frictions, weighted by total earnings in each occupation in 2010. The right panel shows the variance of the log frictions, weighted in the same way.

frictions for black women.

5. Main Results

We are now ready to back out the driving forces behind changes in group occupations and wages every 10 years from 1960 to 2010. Our main focus is the role of changing human capital frictions ($\tau^h$) and labor market discrimination ($\tau^w$) in explaining aggregate U.S. productivity growth during the last half century. However, our approach also allows us to examine how changing preferences for working in different occupations ($z$) and talent for the home sector ($T^{home}$) has affected productivity growth. Human capital frictions, labor market discrimination and occupational preferences vary across occupation-groups in each year while talent in the home sector varies across groups.
Figure 8: Means and Variances of Frictions for Black Women

Note: The left panel shows the average level of the frictions, weighted by total earnings in each occupation in 2010. The right panel shows the variance of the log frictions, weighted in the same way.

and (in a common way across groups) across time. The human capital frictions and occupational preferences remain constant over the lifecycle of a given cohort.

Table 1 summarizes the parameters and normalizations. Table 2 gives the endogenous variables in the model and the target data we used for their indirect inference. For our base specification, we set average ability to be the same across groups in each occupation outside of the home sector ($T_{ig}/T_{i,wm} = 1$). Across groups within an occupation, we think the natural starting point is no differences in mean ability in any occupation; this assumption will be relaxed in our robustness checks. Differences in average ability across occupations are isomorphic to differences in the production technology $A_i$, so we further set $T_{i,wm} = 1$ in all occupations. Again, we do allow average ability in the home sector ($T_{i,home}^g$) to vary across groups, as well as across time in a way that is common across all groups, to help fit labor force participation in the data.

In our base specification, we also assume $\tau_{i,wm}^h = 0$ and $\tau_{i,wm}^w = 0$ so that white men face no occupational barriers in either the labor market or in their human capital decisions. We will consider robustness to instead assuming $\tau_{i,ww}^h = 0$ and $\tau_{i,ww}^w = 0$, so that white women face no barriers. The distinction is whether white women and blacks are discriminated against, or instead preferential treatment is given to white males. We also normalize the occupational preferences of white men such that $z_{i,wm} = 1$ in each occupation. This implies that $z_{i,ww}$ is the relative preference of white women in occupation $i$ relative to white men.

In each year, we have $11 \cdot M + 5$ variables to be determined within the model. For each of the $M$ occupations these are $A_i$, $\phi_i$, $z_{ig}$, $\tau_{i,wm}^h$ and $\tau_{i,wm}^w$. Those with $g$ subscripts...
Table 1: Baseline Parameter Values and Variable Normalizations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Determination</th>
<th>Value</th>
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<td>Fréchet shape</td>
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<td>$\eta$</td>
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<td>Education spending</td>
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<td>$\sigma$</td>
<td>EoS across occupations</td>
<td>Arbitrary</td>
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<td>$\beta$</td>
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<td>Talent in market occupations</td>
<td>Normalized</td>
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<tr>
<td>$z_{i,wm,t}$</td>
<td>Occupational preferences</td>
<td>Normalized</td>
<td>1</td>
</tr>
<tr>
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<td>Human capital barriers</td>
<td>Normalized</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_{i,wm,t}^w$</td>
<td>Labor market barriers</td>
<td>Normalized</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1960 split of $\tau$ into $\tau^h, \tau^w$</td>
<td>Stable split 1960–2010</td>
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Table 2: Endogenous Variables and Empirical Targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Empirical Target</th>
</tr>
</thead>
<tbody>
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<td>Occupations of the young</td>
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<tr>
<td>$\tau_{i,g,t}^w$</td>
<td>Labor market barriers</td>
<td>Wages of the young</td>
</tr>
<tr>
<td>$\phi_{i,t}$</td>
<td>Time elasticity of human capital</td>
<td>Average wages by occupation</td>
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<tr>
<td>$z_{i,g,t}$</td>
<td>Occupational preferences</td>
<td>Wage gaps by occupation</td>
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<td>$A_{i,t}$</td>
<td>Technology by occupation</td>
<td>Wage bill by occupation</td>
</tr>
<tr>
<td>$T_{home,g}$</td>
<td>Talent in the home sector (group)</td>
<td>Labor force participation (group)</td>
</tr>
<tr>
<td>$T_{home,t}$</td>
<td>Talent in the home sector (time)</td>
<td>Labor force participation (time)</td>
</tr>
</tbody>
</table>

Note: The variable values are chosen jointly to match the empirical targets.
have four versions, one each for white men, white women, black women, and black men. This would imply $14 \cdot M$ variables. However, as discussed above, we make three normalizations for white men such that $\tau_{h,wm}^{i} = 1$, $\tau_{w,wm}^{i} = 1$, and $z_{i,wm} = 1$ in each occupation. This leaves us with $11 \cdot M$ variables to be determined within the model. We also allow population shares of each group $q_{g}$ to vary by year to match the data. This gives us an additional 4 variables to be determined each year. Finally, we allow the aggregate efficiency of human capital in the home sector, $T_{t}^{\text{home}}$, to vary across time in a common way across groups. This gives us a fifth variable to be determined in each year.

To identify the values of the $11 \cdot M + 5$ forcing variables in each year, we try to match the following $24 \cdot M - 11$ moments in the data, decade by decade (numbers in parentheses denote the number of moments in each year):

1. The fraction of people from each group-cohort working in each occupation, $p_{ig}(t,c)$ (four groups, three cohorts, and the $p_{ig}(t,c)$ sum to one for each cohort-group).
2. The average wage in each market occupation for each group-cohort.
3. Estimating $\phi_{farm}$ using observed schooling data.

The $A_{i}$ levels and the relative $\phi_{i}$'s across occupations involve the general equilibrium solution of the model, but the intuition for what pins down their values is clear. The level of $A_{i}$ helps determine the overall fraction of the population that works in each occupation. And $\phi_{i}$ is the a determinant of average wage differences across occupations. Thus, the data on employment shares and wages by occupation help pin down the values of $A_{i}$ and $\phi_{i}$. From white men’s wages in each occupation, we can infer the relative values of $\phi_{i}$ across occupations. But we cannot pin down the $\phi_{i}$ levels, as absolute wage levels are also affected by the $A_{i}$ productivity parameters. Thus we need one final normalization to pin down the levels. We choose to match schooling in the lowest wage occupation, which is Farm Non-Managers (in most years). There is a direct link between implied schooling levels associated with occupation $i$ and $\phi_{i}$.$^{31}$ With an estimate of $\phi_{farm}$, we can use average wages in each occupation to recover the

$^{31}$We set $\phi_{farm}$ in a given year to match the observed average schooling among Farm Non-Managers in the same year: $\phi_{farm} = \frac{1 - \eta}{\beta \cdot \frac{s_{farm}}{1 - s_{farm}}}$. This is the only schooling data we use to discipline the estimation. However, in the next section, we will use data on average schooling levels for each group-year to assess how well the model does in explaining changes in schooling levels over time.
remaining $\phi$’s. Again, Table 1 summarizes the parameters and Table 2 the endogenous variables and empirical targets.

5.1. Productivity Gains

Given our model, parameter values, and the forcing variables we infer from the data, we can now answer the key question of the paper: how much of overall growth from 1960 to 2010 can be explained by the changing labor market outcomes of blacks and women during this time period?

Deflating by the NIPA Personal Consumption Deflator, real earnings in the census data grew by 1.3 percent per year between 1960 and 2010.\textsuperscript{32} How much of this growth is due to changing $\tau$’s, according to our model? We answer this question by holding the $A$’s (productivity parameters by occupation), $\phi$’s (schooling parameters by occupation), $z$’s (preferences for each occupation-group) and $q$’s (group shares of the working population) constant over time and letting the $\tau$’s change.\textsuperscript{33}

The results of this calculation are shown in the first column of Table 3. The changes in $\tau$’s account for 30\% to 35\% of growth from 1960 to 2010, whether defined in terms of GDP per person, GDP per worker, earnings per worker, or consumption per person. The second and third columns show that falling barriers to human capital accumulation ($\tau^h$’s) play a much bigger role than falling labor market barriers ($\tau^w$’s), especially for GDP per worker. Specifically, the declining barriers to human capital barriers explain between 75\% and 100\% of the combined effects of $\tau^h$ and $\tau^w$.

A related calculation is to hold the $\tau$’s constant and calculate growth due to changes in the $A$’s, $\phi$’s, and $q$’s. Figure 9 does this. The vast majority of growth in GDP per person is due to increases in $A_i$ and $\phi_i$ over time, but an important part is attributable to reduced frictions. Allowing the $\tau$’s to change as they did historically raises output by 29.8\% between 1960 and 2010.\textsuperscript{34}

\textsuperscript{32}These earnings omit employee benefits.

\textsuperscript{33}We follow the standard approach of chaining. For example, we compute growth between 1960 and 1970 allowing the $\tau$’s to change but holding the other parameters at their 1960 values. Then we compute growth between 1960 and 1970 from changing $\tau$’s holding the other parameters at their 1970 values. We take the geometric average of these two estimates of growth from changing $\tau$’s. We do the same for other decadal comparisons (1970 to 1980 and so on) and cumulate the growth to arrive at an estimate for our entire sample from 1960–2010.

\textsuperscript{34}This absolute growth of 29.8\% is close to the share of growth explained, 30.1\% in Table 3, because earnings per worker grew close to 100\%.
Table 3: Share of Growth due to Changing Frictions

<table>
<thead>
<tr>
<th>—— Share of growth accounted for by ——</th>
<th>Full Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^h$ and $\tau^w$</td>
<td>$\tau^h$ only</td>
</tr>
<tr>
<td>GDP per person</td>
<td>30.1%</td>
</tr>
<tr>
<td>GDP per worker</td>
<td>35.4%</td>
</tr>
<tr>
<td>Earnings</td>
<td>33.0%</td>
</tr>
<tr>
<td>Consumption</td>
<td>31.6%</td>
</tr>
<tr>
<td>LF Participation</td>
<td>12.4%</td>
</tr>
</tbody>
</table>

Note: Entries in the table show the share of growth in the model attributable to changing frictions under various assumptions. The variables are $\tau^h$ (human capital frictions), $\tau^w$ (labor market frictions), $z$ (occupational preference), and $T_{g}^{home}$ (home sector ability), as discussed in the text. The last column reports the share of observed growth explained by the full model solution, including all of these variables plus $A, \phi$ and $q$.

Returning to Table 3, the changing $\tau$'s explain less of the trend in overall labor-force participation – only about 12%. There are two offsetting factors. Falling barriers in the labor market ($\tau^w$) do account for about one-third of the rise in women's participation. But falling barriers to human capital accumulation ($\tau^h$) go the other way, because they occurred disproportionately in sectors where we infer a low participation rate. Changes in $T_{g}^{home}$, on the other hand, explain more than 100% of the increase in labor force participation during this time period. In order to explain participation rates and GDP per worker it is essential to account for changes in $T_{g}^{home}$.

If one compares the first and fourth columns of Table 3, one can see very modest effects from changing preferences for each occupation among each group ($z_{ig}$'s). Why can't changing preferences for market work explain women's rising labor force participation relative to white men? If women simply did not like some occupations, they should be paid more in occupations in which they are underrepresented. We observe no such tendency in the data, either in levels or in changes. The gender gap in wages was no lower in skilled occupations, and it did not fall faster in skilled occupations as women's representation rose. It is these facts that allow us to infer that changing
preferences had little effect on aggregate productivity growth during the last fifty years.

The next-to-last row in Table 3 implies a major impact of falling $T_{g}^{home}$ (usefulness of human capital in the home sector) on labor force participation and, in turn, GDP per worker. According to the model, the falling $T_{g}^{home}$ values raised GDP per capita but lowered GDP per worker by attracting workers with less of a comparative advantage in the market sector. For the same reason, the model predicts that the gender wage gap fell despite the falling $T_{g}^{home}$'s.

The last row in Table 3 reflects the role of changing $A_i$'s and $\phi_i$'s. Trends in productivity and skill-intensity account for the bulk of growth in GDP per person, GDP per worker, earnings, and consumption. This drives home that the model does not require changing barriers to generate growth. The technology trends hold down labor force participation because they favor occupations where we infer low participation rates.

Table 4 shows how the changing $\tau$'s affect wage gaps and earnings across groups. For women, the changing $\tau$'s, in particular the changing $\tau^h$'s, more than explain the shrinking gender gap in wages. The model says that, in the absence of changing $\tau$'s, the rising labor force participation rate of women would have widened the gender gap.
Table 4: Wages by Group and Changing Frictions

<table>
<thead>
<tr>
<th></th>
<th>—— Share of growth accounted for by ——</th>
<th>Full Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau^h$ and $\tau^w$</td>
<td>$\tau^h$ only</td>
</tr>
<tr>
<td>Wage gap, WW</td>
<td>211.5%</td>
<td>174.9%</td>
</tr>
<tr>
<td>Wage gap, BM</td>
<td>93.3%</td>
<td>77.6%</td>
</tr>
<tr>
<td>Wage gap, BW</td>
<td>147.1%</td>
<td>116.6%</td>
</tr>
<tr>
<td>Earnings, WM</td>
<td>0.6%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Earnings, WW</td>
<td>73.1%</td>
<td>49.2%</td>
</tr>
<tr>
<td>Earnings, BM</td>
<td>25.3%</td>
<td>23.1%</td>
</tr>
<tr>
<td>Earnings, BW</td>
<td>50.2%</td>
<td>34.5%</td>
</tr>
</tbody>
</table>

Note: Entries in the table show the share of growth in the model attributable to changing frictions under various assumptions. The frictions are $\tau^h$ (human capital), $\tau^w$ (labor market), $z$ (occupational preference), and $T_{home}^g$ (labor force participation), as discussed further in the text. The last column reports the share of observed growth explained by the full model solution, including the frictions as well as the $A$ and $\phi$ parameters.

by bringing in women with less of a comparative advantage in market occupations (compared to other women in the market, not men).

The changing $\tau$’s account for little wage growth for white men, not surprisingly. But they do account for 73% of earnings growth for white women, 25% for black men, and 50% for black women. Again, changing $\tau^h$’s play a larger role than changing $\tau^w$’s. For both black men and white men, wage growth was driven primarily by changes in technology and skill requirements ($A$’s and $\phi$’s).

5.2. Robustness

Table 5 explores the robustness of our results to alternative counterfactuals. The first row contains benchmark results for comparison. The next two rows show that the productivity gains we estimate are not driven by the gender and race wage gaps we feed into the model. We can halve the wage gaps in all years, or even eliminate them in
Table 5: Robustness to Alternative Counterfactuals

<table>
<thead>
<tr>
<th></th>
<th>( \tau^h ) and ( \tau^w )</th>
<th>( \tau^h ) only</th>
<th>( \tau^w ) only</th>
<th>( \tau^h, \tau^w, z )</th>
<th>( \tau^h, \tau^w, z, T_{g}^{home} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>30.1%</td>
<td>27.0%</td>
<td>3.1%</td>
<td>30.8%</td>
<td>36.6%</td>
</tr>
<tr>
<td>Wage gaps halved</td>
<td>30.3%</td>
<td>26.0%</td>
<td>4.3%</td>
<td>30.9%</td>
<td>38.6%</td>
</tr>
<tr>
<td>Zero wage gaps</td>
<td>30.5%</td>
<td>24.6%</td>
<td>5.9%</td>
<td>30.9%</td>
<td>40.3%</td>
</tr>
<tr>
<td>No frictions in</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“brawny” occupations</td>
<td>28.1%</td>
<td>24.4%</td>
<td>3.7%</td>
<td>29.2%</td>
<td>34.0%</td>
</tr>
<tr>
<td>No frictions in 2010</td>
<td>32.6%</td>
<td>30.2%</td>
<td>2.5%</td>
<td>33.4%</td>
<td>39.5%</td>
</tr>
</tbody>
</table>

Note: See notes to Table 3. In the fourth line, we assume that there are no frictions for white women in occupations where physical strength is important. Instead, we allow \( T_{i,ww} \) to change over time to match the occupational allocation for white women. For blacks in this case, we do allow for frictions, but also assume \( T_{i,bw} = T_{i,ww} \). “No frictions in 2010” assumes that there are no frictions in 2010, so that \( T_{i,g} \) differences explain all group differences in that year; we then calculate \( \tau \)’s for earlier years assuming the \( T \)’s for 2010 apply to earlier years.

all years, and the implied \( \tau \)’s explain basically the same 30% of growth as in the baseline. One reason is that misallocation of talent by race and gender can occur even if average wages are similar. The misallocation of talent is tied to the dispersion in the \( \tau \)’s, whereas the wage gaps are related to both the mean and variance of the \( \tau \)’s. Another reason is that we think the wage gaps would have widened due to the changing \( A \)’s and \( \phi \)’s in the absence of the changing \( \tau \)’s. The upshot is that model productivity gains cannot be gleaned from the wage gaps alone.

The next row in Table 5 relaxes the assumption that men and women draw from the same distribution of talent in all occupations. In particular, we consider the possibility that some occupations rely more on physical strength than others, and that this reliance might have changed because of technological progress. To see the potential importance of this story, we go to the extreme of assuming no frictions faced by white women in any of the occupations where physical strength is arguably important. These occupations include construction, firefighters, police officers, and most of manufac-
We estimate values for $T_{ig}$ for white women that fully explain the allocation of young women to these occupations in 1960, 1970, . . . , 2010. As shown in Table 5, the fraction of growth explained by changing frictions falls modestly from 30% to 28% if we assume that all the movement of women into manufacturing, construction, police, firefighting and other brawny occupations was due to changes in relative comparative advantages in these occupations as opposed to changing $\tau$’s in these occupations. Thus most of the productivity gains come from the rising propensity of women to become lawyers, doctors, scientists, professors, and managers – occupations where physical strength is not important.

We next assume that all group differences among the young in 2010 reflect talent rather than distortions. I.e., we set 2010 $\tau$’s to zero and impose values of $T_{ig}$ to fully account for group differences among the young in 2010. We keep talent in previous years at the 2010 values for each group, but back out distortions in earlier years. Surprisingly, eliminating $\tau$’s in the earlier years generates somewhat larger productivity gains (32.6%) than in the baseline case (30.1%).

Table 6 explores robustness of our productivity gains to different parameter values. For each set of parameter values considered, we recalculate the $\tau$, $z$, $T_{home}$, $A$, and $\phi$ values so that the model continues to fit the occupation shares, wage gaps, etc. The first row of Table 6 replicates the gains under baseline parameter values for comparison. The next two rows consider higher values of the Fréchet shape parameter $\theta$, which is inversely related to the dispersion of comparative advantage across occupations. Recall that our baseline $\theta$ was estimated from wage dispersion within occupation-groups. This may overstate the degree of comparative advantage, as some wage variation may be due to absolute advantage. With less comparative advantage, it matters less who is allocated to which occupations. We thus entertain higher values of $\theta$ of 4 and 7. As shown in Table 6, the productivity gains from changing $\tau$’s do fall, from 30% with baseline $\theta = 2.12$ to 26% and 24%, respectively, with $\theta = 4$ and $\theta = 7$. As discussed above, such high values of $\theta$ imply correspondingly higher labor supply elasticities, since comparative advantage in the market sector is weaker with higher $\theta$. Values of $\theta$ of 4 or lower are needed to match the extensive margin labor supply elasticities estimated for men and women.

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35Rendall (2010) classifies occupations based on the importance of physical strength. We define brawny occupations as those below the median in Rendall (2010).
Table 6: Robustness to Parameter Values

<table>
<thead>
<tr>
<th>—— GDP per person growth accounted for by ——</th>
<th>$\tau^h$ and $\tau^w$</th>
<th>$\tau^h$ only</th>
<th>$\tau^w$ only</th>
<th>$\tau^h$, $\tau^w$, $z$</th>
<th>$\tau^h$, $\tau^w$, $z$, $T^b_{\text{home}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>30.1%</td>
<td>27.0%</td>
<td>3.1%</td>
<td>30.8%</td>
<td>36.6%</td>
</tr>
<tr>
<td>$\theta = 4$</td>
<td>26.2%</td>
<td>20.9%</td>
<td>5.3%</td>
<td>29.0%</td>
<td>40.5%</td>
</tr>
<tr>
<td>$\theta = 7$</td>
<td>24.4%</td>
<td>17.5%</td>
<td>8.0%</td>
<td>31.6%</td>
<td>44.3%</td>
</tr>
<tr>
<td>$\eta = 1/6$</td>
<td>27.5%</td>
<td>23.8%</td>
<td>3.8%</td>
<td>28.7%</td>
<td>38.4%</td>
</tr>
<tr>
<td>$\eta = 1/2$</td>
<td>33.1%</td>
<td>30.2%</td>
<td>3.0%</td>
<td>33.3%</td>
<td>34.5%</td>
</tr>
<tr>
<td>$\sigma = 1.05$</td>
<td>30.1%</td>
<td>27.0%</td>
<td>3.0%</td>
<td>30.7%</td>
<td>36.5%</td>
</tr>
<tr>
<td>$\sigma = 10$</td>
<td>30.2%</td>
<td>27.0%</td>
<td>3.2%</td>
<td>30.9%</td>
<td>36.7%</td>
</tr>
</tbody>
</table>

Note: See notes to Table 3. The baseline parameter values are $\theta = 2.12$, $\eta = 0.358$, and $\sigma = 3$.

Table 6 also varies $\eta$, the elasticity of human capital with respect to goods invested in human capital. As one might expect, the higher the value of $\eta$, the bigger the gains from reducing human capital and labor market frictions: the gains rise from 27.5% with $\eta = 1/6$ to 30% with our baseline $\eta = 0.358$ to 33% with $\eta = 1/2$. Recall that $\eta$ should be the share of output invested in human capital, and its baseline value was set to match education spending as a share of labor earnings.

We next report sensitivity to the elasticity of substitution ($\sigma$) between occupations in production. As shown in Table 6, the gains to changing $\tau$’s are very similar with $\sigma = 1.05$ (close to Cobb-Douglas aggregation) and with $\sigma = 10$ as with the baseline of $\sigma = 3$.

Although not shown in the robustness tables, the gains are not at all sensitive to $\beta$, the weight placed on time vs. goods in utility. The gains do not change to one decimal point as we move its value from 0.5 to 0.8 around the baseline value of $\beta = 0.693$. The moderate sensitivity of our results to $\theta$, $\eta$, $\sigma$ and $\beta$ may seem puzzling. But note that, as we entertain different parameter values, we simultaneously change the baseline $A$’s and $\tau$’s to fit observed wages and employment shares for each occupation and group in each year.
Table 7: More Robustness

<table>
<thead>
<tr>
<th></th>
<th>( \tau^h ) and ( \tau^w )</th>
<th>( \tau^h ) only</th>
<th>( \tau^w ) only</th>
<th>( \tau^h, \tau^w, z )</th>
<th>( \tau^h, \tau^w, z, T^h_{\text{home}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>30.1%</td>
<td>27.0%</td>
<td>3.1%</td>
<td>30.8%</td>
<td>36.6%</td>
</tr>
<tr>
<td>Less home experience</td>
<td>30.1%</td>
<td>7.2%</td>
<td>2.9%</td>
<td>30.8%</td>
<td>36.3%</td>
</tr>
<tr>
<td>LFP minimum factor = 1/3</td>
<td>30.4%</td>
<td>27.2%</td>
<td>3.2%</td>
<td>31.2%</td>
<td>36.8%</td>
</tr>
<tr>
<td>LFP minimum factor = 2/3</td>
<td>30.1%</td>
<td>26.9%</td>
<td>3.2%</td>
<td>30.7%</td>
<td>36.6%</td>
</tr>
<tr>
<td>evalmiddle WeightPig=1</td>
<td>30.3%</td>
<td>27.0%</td>
<td>3.3%</td>
<td>31.2%</td>
<td>36.7%</td>
</tr>
<tr>
<td>evalmiddle WeightPig=0</td>
<td>30.3%</td>
<td>27.2%</td>
<td>3.1%</td>
<td>30.5%</td>
<td>37.1%</td>
</tr>
<tr>
<td>Split of ( \hat{\tau} ) in 1960 to ( \tau^w = 1/2 )</td>
<td>28.9%</td>
<td>16.8%</td>
<td>12.2%</td>
<td>30.0%</td>
<td>32.8%</td>
</tr>
<tr>
<td>50/50 split of ( \hat{\tau}_{i,g} )</td>
<td>31.0%</td>
<td>23.6%</td>
<td>7.5%</td>
<td>33.1%</td>
<td>36.1%</td>
</tr>
</tbody>
</table>

Note: See notes to Table 3. The baseline parameter values are \( \theta = 2.12 \), \( \eta = 0.358 \), and \( \sigma = 3.12 \). “WeightPig=1” in evalmiddle puts all the weight on matching \( p_{\text{ig}} \) for the middle-age group, while “WeightPig=0” puts all the weight on matching wage growth; the benchmark case has WeightPig=1/2.
6. Further Model Implications

While our model may appear stylized in many respects, it does very well at matching many additional empirical facts. In this section, we show that our model matches estimates of trends in female labor supply elasticities over time, cross state variation in survey measures of racial discrimination, and time series trends in schooling for men and women.

6.1. Trends in Female Labor Supply Elasticities

Using data from the Current Population Survey, Blau and Kahn (2007) estimate that during the 1980-2000 period there was a dramatic decline in female labor supply elasticity. Their analysis is well suited to compare to the predictions of our model in that they report female labor supply elasticities for 25-34 year olds, 35-44 year olds, and 45-54 year old in 1980, 1990, and 2000. They estimate their elasticities by regressing hours worked on wages controlling for other observable factors (like husbands earnings, assets, number of children) and adjusting for missing wages of the nonemployed and measurement error. They find that labor supply elasticities of women (with respect to changes in their own wage) fell by roughly 50 percent during the 1980-2010 period.

Figure 10 plots the estimates of female labor supply elasticities from Blau and Kahn for various age-year groups against the models implied labor supply elasticities for white women for the same age-year groups. Each point on the plot is a age-year group. In total, there are nine points: one for the younger, middle age, and older age groups in the years 1980, 1990, and 2000. For the Blau and Khan estimates, we use the results from their Model 1 reported in their Table 2. As seen from Figure 10, the implied labor supply elasticities for white women from our model tracks very closely the estimated labor supply elasticities for married women found by Blau and Khan. Consider women aged 45-54 in 1980 and 2000. Blau and Kahn estimate labor supply elasticities for these two groups of 1.1 and 0.5, respectively. Our model implies labor supply elasticities for these groups of 2.2 and 1.0, respectively. Our estimates are systematically slightly higher than the Blau and Khan estimates - potentially do to the fact that we are focusing on all white women as opposed to married women regardless of race. But, the key

36They restrict their sample to include only married women.
finding we want to highlight is that the implied decline in labor supply elasticities from our model is nearly identical to the estimated decline in labor supply elasticities documented by Blau and Kahn. For example, in our model, women aged 45-55 experienced a decline in labor supply elasticities between 1980 and 2000 of 55 percent. Blau and Khan also report a decline in labor supply elasticities for this group of 55 percent during the same time period. The results are not just concentrated among the older group. Between 1980 and 2000, our model matches nearly identically the percentage decline in labor supply elasticities for the young and middle age groups as documented in Blau and Kahn (2007).

It should be noted that nothing in our model is calibrated to match either the level or the trend in labor supply elasticities for women. As discussed above, we picked our estimate of $\theta$ to match the labor supply elasticity of men in 1980. With that parameter pinned down, our model implies that women’s labor supply elasticity is only a function of female labor force participation. The fact that we can roughly match the level of female labor supply as well as the trend for different cohorts shows that our model is consistent with empirical moments outside of the ones used to calibrate the model.
6.2. Cross State Measures of Discrimination

Among other things, our measures of $\tau$s can be thought as proxying for measures of discrimination against blacks and women. There are very few micro based measures of discrimination to which we can compare our estimated $\tau$s. One such exception is the recent work by Charles and Guryan (2008). Charles and Guryan (CG) used data from the General Social Survey (GSS) to make a measure of the taste for discrimination for every state. The GSS asks a large nationally representative sample of individuals about their views on a variety of issues. A series of questions have been asked over the years assessing the respondents attitudes towards race. For example, questions asked about individuals views on cross-race marriage, school segregation, and the ability for homeowners to discriminate with respect to home sales. Pooling together survey questions from the mid 1970s through the early 1990s and focusing only a sample of white respondents, Charles and Guryan make indices of the extent of racial discrimination for each state. We focus on their marginal discrimination measure.

For each state, they create a standardized discrimination index such that it has mean zero and a standard deviation of 1. For essentially all states, the index is below zero given they are focusing on the taste for discrimination for the marginal person where the marginal person corresponds to the fraction of the workforce that is black.\[^{37}\] Higher values of the CG discrimination measure imply more discrimination. They compute their measure for 44 states.

Using our model above, we can make a composite tau measure of black men relative to white men for each U.S. state. When making a composite tau of black men relative to white men, we make a few simplifying assumptions to ensure we have enough power for each state. First, we assume that black and white men have the same preferences for each occupation (i.e., $z_{i,wm} = z_{i,bm}$ for all $i$). Second, we assume that there are no cohort effects in our composite measure of $\tau$. This latter assumption allows us to pool together all cohorts within a year when computing our measure of $\tau$. With these assumptions, we can compute for each state a measure of $\tau_{i,bm}$.

Again, we do not have enough power to measure $\tau_{i,bm}$ for each state and each year.

\[^{37}\]The concept of the marginal discriminator comes from Becketts theory of discrimination. If there are 10 percent of blacks in the state labor market, it is only the discrimination preferences of the white person at the 10th percentile of the white distribution that matters for outcomes (with the first percentile being the least discriminatory).
using our 67 occupations. To further help with power, we collapsed our 67 occupations to 20 occupations (excluding the home sector). Appendix Table X talks about the reduced grouping of occupations. Additionally, we pooled together data from 1980 and 1990 to make a measure of $\tau_{i,bm}$ based on data for both the 80s and 90s. We do this because the CG discrimination measure is based on data pooled from the GSS between 1977 and 1993. For each state, we will have 20 different measures of $\tau_{i,bm}$. We aggregate the $\tau_{i,bm}$ to one measure of $\tau_{bm}$ for each state by taking a weighted average of the occupation level $\tau_s$ where the weights are based on share of the occupations income (for the country as whole) out of total income across all occupations (for the country as a whole). Lastly, we exclude states with an insufficient amount of black households to compute our measure of $\tau_{bm}$. Given the CG restrictions from the GSS and our restrictions from the Census data, we are left with 37 states.

Figure 11 shows a simple scatter plot between the CG measure of discrimination and our measure $\tau_{bm}$. Each observation in the scatter plot is a U.S. state where the size of the circle represents the number of black men within our Census sample. We also show the weighted OLS regression line on the figure. As seen from the figure, there is a very strong relationship between our measures of $\tau_{bm}$ and the CG discrimination index. The adjusted R-squared of the simple scatter plot is 0.6 and the slope of the regression line is 0.81 with a standard error of 0.11. Places we identify as having a high $\tau_{bm}$ are the same places Charles and Gurley find as being highly discriminatory based on survey data from the GSS. The findings in Figure 11 provide additional external validity that our procedure is measuring salient features of U.S. economy over the last five decades.

6.3. Trends in Schooling

[To be completed]

6.4. Conclusion

How does discrimination in the labor market and in the acquisition of human capital affect occupational choice? And what are the consequences of the resulting allocation of talent for aggregate productivity? We develop a framework to tackle these questions empirically. This framework has three building blocks. First, we use a standard Roy model of occupational choice, augmented to allow for labor market discrimination and
Figure 11: Model implied $\tau$’s for Black Men vs. Survey Measures of Discrimination, by U.S. State

Note: Figure plots measures of our model’s implied composite $\tau$’s for black men for each state using pooled data from the 1980 and 1990 census (x-axis) against survey based measures of discrimination against blacks for each state as reported in Charles and Guryan (2008). The Charles and Guryan data are complied using data from the General Social Survey between 1977 and 1993. We use their marginal discrimination measure for this figure. See text for additional details.
discrimination in the acquisition of human capital. Second, we impose the assumption that the distribution of an individuals ability over all possible occupations follows an extreme value distribution. Third, we embed the Roy model in general equilibrium to account for the effect of occupational choice on the price of skills in each occupation, and to allow for the effect of technological change on occupational choice.

We apply this framework to measure the changes in barriers to occupational choice facing women and blacks in the U.S. from 1960 to 2010. We find large reductions in these barriers, concentrated in high-skilled occupations. We then use our general equilibrium setup to isolate the aggregate effects of the reduction in occupational barriers facing these groups. Our calculations suggest that falling barriers may explain one-third of aggregate growth in output per worker. Declining barriers to human capital accumulation play a larger role than falling labor market barriers.

It should be clear that this paper provides only a preliminary answer to these important questions. The general equilibrium Roy model we use is a useful place to start, but it could be a poor approximation of the U.S. labor market.

We have focused on the gains from reducing barriers facing women and blacks over the last fifty years. But we suspect that barriers facing children from less affluent families and regions have worsened in the last few decades. If so, this could explain both the adverse trends in aggregate productivity and the fortunes of less-skilled Americans in recent decades. We hope to tackle some of these questions in future work.

References


Online Appendix to “The Allocation of Talent and U.S. Economic Growth”
(Not for publication)
Chang-Tai Hsieh, Erik Hurst, Charles I. Jones, Peter J. Klenow
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A Derivations and Proofs

The propositions in the paper summarize the key results from the model. This appendix shows how to derive the results.

Proof of Proposition 1. Occupational Choice

As given in equation (7), the individual’s utility from choosing a particular occupation, \( U(\tau_i, w_i, \epsilon_i) \), is proportional to \( (\tilde{w}_{ig}\epsilon_i)^{\beta_{1-\eta}} \), where
\[
\tilde{w}_{igc} \equiv \frac{T_{ig}(c)w_{ig}(c)s_{ig}(c)[(1-s_{ig}(c))\tilde{z}_{ig}(c)]^{1-\eta}}{\tau_{ig}(c,c)}
\]
and where, suppressing the occupation/group subscripts, \( \bar{T}(c) \equiv \frac{1}{3}(T_y + T_m + T_o) \) is the average of the lifetime experience terms. The solution to the individual’s problem, then, involves picking the occupation with the largest value of \( \tilde{w}_{ig}\epsilon_i \). To keep the notation simple, we will suppress the \( g \) subscript in what follows.

Without loss of generality, consider the probability that the individual chooses occupation 1, and denote this by \( p_1 \). Then

\[
p_1 = \Pr[\tilde{w}_1\epsilon_1 > \tilde{w}_s\epsilon_s] \forall s \neq 1
\]
\[
= \Pr[\epsilon_s < \tilde{w}_1\epsilon_1/\tilde{w}_s] \forall s \neq 1
\]
\[
= \int F_1(\epsilon, \alpha_2\epsilon, \ldots, \alpha_N\epsilon)d\epsilon,
\]
(28)

where \( F_1(\cdot) \) is the derivative of the cdf with respect to its first argument and \( \alpha_i \equiv \tilde{w}_1/\tilde{w}_i \).

Recall that
\[
F(\epsilon_1, \ldots, \epsilon_N) = \exp \left[ \sum_{s=1}^{N} \epsilon_s^{\theta} \right].
\]

Taking the derivative with respect to \( \epsilon_1 \) and evaluating at the appropriate arguments gives
\[
F_1(\epsilon, \alpha_2\epsilon, \ldots, \alpha_N\epsilon) = \theta\epsilon^{\theta-1} \cdot \exp \left[ \tilde{\alpha}\epsilon^{-\theta} \right].
\]
(29)
where \( \bar{\alpha} \equiv \sum_s \alpha_s^{-\theta} \).

Evaluating the integral in (28) then gives

\[
p_1 = \int F_1(\epsilon, \alpha_2 \epsilon, \ldots, \alpha_N \epsilon) d\epsilon = \frac{1}{\bar{\alpha}} \int \bar{\alpha} \theta \epsilon^{-\theta - 1} \cdot \exp \left[ \bar{\alpha} \epsilon^{-\theta} \right] d\epsilon = \frac{1}{\bar{\alpha}} \cdot \int dF(\epsilon) = \frac{1}{\bar{\alpha}} = \frac{1}{\sum_s \alpha_s^{-\theta}} = \frac{\bar{\alpha}^{\theta}}{\sum_s \bar{\alpha}_s^{\theta}}.
\]

A similar expression applies for any occupation \( i \), so we have

\[
p_i = \frac{\bar{\alpha}_i^{\theta}}{\sum_s \bar{\alpha}_s^{\theta}}.
\]

\textbf{Proof of Proposition 3. Average Quality of Workers}

Total efficiency units of labor supplied to occupation \( i \) by group \( g \) are

\[
H_{ig} = q_g p_{ig} \cdot \mathbb{E} [h_i \epsilon_i | \text{Person chooses } i].
\]

Recall that \( h(\epsilon, s) = s^\phi_i e^\eta \). Using the results from the individual's optimization problem, it is straightforward to show that

\[
h_i \epsilon_i = (s_i^\phi_i)^{\frac{1}{1-\eta}} \left( \eta \omega_i (1 - \tau_i^w) (T_y + T_m + T_o) \right)^{\frac{\eta}{1-\eta}} \frac{1}{\epsilon_i^{\frac{1}{1-\eta}}}.
\]

Therefore,

\[
H_{ig} = q_g p_{ig} (s_i^\phi_i)^{\frac{1}{1-\eta}} \left( \eta \omega_i (1 - \tau_i^w) (T_y + T_m + T_o) \right)^{\frac{\eta}{1-\eta}} \cdot \mathbb{E} \left[ \epsilon_i^{\frac{1}{1-\eta}} | \text{Person chooses } i \right].
\]

(30)

To calculate this last conditional expectation, we use the extreme value magic of the
Fréchet distribution. Let \( y_i \equiv \tilde{w}_i \epsilon_i \) denote the key occupational choice term. Then

\[
y^* \equiv \max_i \{y_i\} = \max_i \{\tilde{w}_i \epsilon_i\} = \tilde{w}^* \epsilon^*.
\]

Since \( y_i \) is the thing we are maximizing, it inherits the extreme value distribution:

\[
\Pr[ y^* < z ] = \Pr[ y_i < z ] \forall i
\]
\[
= \Pr[ \epsilon_i < z/\tilde{w}_i ] \forall i
\]
\[
= F \left( \frac{z}{\tilde{w}_1}, \ldots, \frac{z}{\tilde{w}_N} \right)
\]
\[
= \exp \left[ - \sum_s \tilde{w}_s^\theta z^{-\theta} \right]
\]
\[
= \exp \{ -mz^{-\theta} \}.
\]

That is, the extreme value also has a Fréchet distribution, where \( m \equiv \sum_s \tilde{w}_s^\theta \).

Straightforward algebra then reveals that the distribution of \( \epsilon^* \), the ability of people in their chosen occupation, is also Fréchet:

\[
G(x) \equiv \Pr[ \epsilon^* < x ] = \exp[ -m^* x^{-\theta} ]
\]

where \( m^* \equiv \sum_{s=1}^N (\tilde{w}_s/\tilde{w}^*)^\theta = 1/\tilde{p}^* \). This result is useful later in the paper in that it implies that the wage distribution across people within an occupation will also be Fréchet, with a parameter that depends on \( \theta \).

Finally, one can then calculate the statistic we needed above back in equation (30): the expected value of the chosen occupation’s ability raised to some power. In particular, let \( i \) denote the occupation that the individual chooses, and let \( \lambda \) be some positive exponent. Then,

\[
\mathbb{E}[\epsilon_i^\lambda] = \int_0^\infty \epsilon^\lambda dG(\epsilon)
\]
\[
= \int_0^\infty \theta m^* e^{-\theta + \lambda} e^{-m^* \epsilon^{-\theta}} d\epsilon
\]

Recall that the “Gamma function” is \( \Gamma(\alpha) \equiv \int_0^\infty x^{\alpha-1} e^{-x} dx \). Using the change-of-variable
\( x \equiv m^* \epsilon^{-\theta} \), one can show that
\[
E[\epsilon^\lambda] = m^* \lambda/\theta \int_0^\infty x^{-\lambda/\theta} e^{-x} dx
\]
\[
= m^* \lambda/\theta \Gamma \left( 1 - \frac{\lambda}{\theta} \right).
\]  
(34)

Applying this result to our model, we have
\[
E \left[ \epsilon_i^{1/\eta} \mid \text{Person chooses } i \right] = m^* \frac{1}{\theta} \frac{1}{1-\eta} \Gamma \left( 1 - \frac{1}{\theta} \cdot \frac{1}{1-\eta} \right)
\]
\[
= \left( \frac{1}{\bar{\eta})} \right)^\frac{1}{\eta} \frac{1}{1-\eta} \Gamma \left( 1 - \frac{1}{\beta} \cdot \frac{1}{1-\eta} \right).
\]  
(35)

Substituting this expression into (30) and rearranging leads to the last result of the proposition.

**Proof of Proposition** 4. Occupational Wage Gaps

The proof of this proposition is straightforward given the results of Proposition 1. Note that \( \bar{\eta} \equiv \eta^{\eta/(1-\eta)} \).

**B Identification and Estimation**

This section explains how we identify and estimation the frictions and other parameters. Need to update this section to include \( \bar{T} \equiv (T_y + T_m + T_o) \).

**B1. Wage and Schooling Time**

We use data from the young white men to recover wage, \( w_i \), and schooling time, \( s_i \), parameters. \( w_i \) denotes the wage per efficiency unit of labor paid by firm for an occupation \( i \). Recall that a person in occupation \( i \) and group \( g \) is paid a wage equal to \( (1 - \tau_i^w) w_i \). \( s_i \) represents time spent on human capital accumulation. From the first order condition, we had the following relationship between schooling time \( s_i \) and the elasticity of human capital with respect to time \( \phi_i \) (and defining \( \beta \equiv 3\bar{\beta} \)):

\[
s_i = \frac{1}{1 + \frac{1-\eta}{\beta \phi_i}}
\]  
(36)
Suppose that frictions for the white men, $\tau_{i,wm}(c, t)$, are given for every cohort $c$ and year $t$. We will calculate these to balance out rents collected from other three groups later. In every year, we use occupational propensities and average wage to infer wage and schooling time parameters.

We assume that elasticity of human capital with respect to time, $\phi_i$, for the farm work are predetermined somehow. At the moment, we are using the elasticity for the farm work from the version 3.0 of the paper.

Given these assumptions on the elasticities, we estimate $w_i$ and $s_i$ (and therefore $\phi_i$) by the following steps:

1. Make a guess on $w_{i, \text{home}}(c)$ for $c = 60, 70, 80, 90, 00, 07$.

2. Calculate $m_{wm}(c)$ from the farm workers, given $\phi_{\text{farm}}(c)$:

$$m_{wm}(c) \equiv \sum_s \tilde{w}_{sg}(c) \theta$$

$$= \left( \frac{\text{wage}_{i,wm}(c, c)(1 - s_i(c))^{1/\beta}}{\gamma \eta^{1/\theta}} \right)^{\theta(1-\eta)} - \left( \frac{T_{i,wm}^{\text{home}} w_{\text{home}} s_i(c)(1 - s_i(c))^{1-\eta}}{1 - \tau_{wm}(c)} \right) \theta$$

3. Solve a non-linear equation of $s_i(c)$ for each time $c$ and occupation $i$. We assume probabilities of working at young, $P_{i,wm}^{\text{work}}(c, c)$, are the same across occupations. So, the probability of working at young for each occupation is the aggregate labor force participation rate of the young. As a normalization, we assume the average level of $T_{i,wm}^{\text{home}}$ to be one for each period.

$$P_{i,wm}(c, c) = P_{i,wm}^{\text{work}}(c, c) \frac{w_i(c) s_i(c)^{\phi_i(c)} (1 - s_i(c))^{1-\eta}}{m_{wm}(c)} \theta$$

where

$$P_{i,wm}^{\text{work}}(c, c) = \frac{1}{1 + \left( \frac{T_{i,wm}^{\text{home}} w_{\text{home}} s_i(c)^{\phi_i(c)} (1 - s_i(c))^{1-\eta}}{1 - \tau_{wm}(c)} \right)^{\theta}}$$

4. Given $s_i(c)$, solve for $w_i(c)$ using occupational propensities:

$$w_i(c) = \left[ \frac{P_{i,wm}(c, c) * m_{wm}(c)}{P_{i,wm}^{\text{work}}(c, c)} \right]^{1/\theta} \frac{1}{s_i(c)^{\phi_i(c)} (1 - s_i(c))^{1-\eta}}$$
5. Repeat steps 1 to 4 until the labor force participation converges to the data by changing the guess on $w_{\text{home}}$.

Implicit wage in the home sector, $w_{\text{home}}$, is a key element. It will govern the labor force participation over time in the later steps. Furthermore, different subsidy rates to white men will alter estimates for this implicit wage. We will come back how we find converged subsidy rate and $w_{\text{home}}$ values. The basic idea is to balance out a deadweight loss from the friction against other three groups with appropriate subsidy rates.

**B2. Frictions from the Cohort Data**

Given wage and schooling time parameters calibrated from the young white men, we estimate the frictions in the labor market and human capital accumulation, $\tau_{ig}^w(c, t)$ and $\tau_{ig}^h(c, t)$, for every cohorts and time. Every cohorts in given year share the same degree of $\tau_{ig}^w$ in the labor market. On the other hands, the friction in schooling, $\tau_{ig}^h$, is cohort-specific. Therefore, we have the following conditions on the friction: (i) $\tau_{ig}^w(\cdot, t)$ are the same for the young, the middle-aged and the old given year and (ii) $\tau_{ig}^h(c, \cdot)$ are the same when the cohort become the middle-aged and the old from the young.

We solve the same problem for three groups, white women, black men, and black women, sequentially. Without loss of generality, we can interpret the group $g$ here as white women.

In every year, we have three cohorts: the young, the middle-aged and the old. We start to solve the problem of the young in 1960 and use estimated parameters to infer frictions for the later cohorts.

First of all, we solve a problem of the young in 1960. We divide the aggregate friction into the labor market and human capital accumulation by the following rule:

\[
\tau_{ig} = \tau_{ig}^\alpha * \tau_{ig}^{(1-\alpha)} = \frac{1}{1-\tau_{ig}^w} * \left(1 + \tau_{ig}^h\right)\eta
\] (40)

As a normalization, we assume the equal dividing rule, $\alpha = 0.5$.

Now, we infer $m_g$, $z_{ig}$, and $T_{ig}^{\text{home}}$ from data on the labor force participation rate, occupational propensities, and average wages. $m_g$ governs the average level of preference terms $z_{ig}$. Assuming the average level to be one, we have the following two equations for each occupation to estimate two parameters, $z_{ig}$ and $T_{ig}^{\text{home}}$. 

\[
z_{ig}(c) = \left[ \gamma^{\frac{1}{1-\eta}} \left( \frac{m_g(c)}{P_{\text{work}}^ig(c,c)} \right)^{\frac{1}{1-\eta}} \frac{1}{\text{wage}_{ig}(c,c)} \right]^\beta \frac{1}{1 - s_i(c)}
\] (41)

\[
P_{\text{work}}^ig(c,c) = 1 - P_{ig}(c,c) \left[ \frac{T_{\text{home}}^ig(c,c) w_{\text{home}}(c,c) s_i(c) \phi_i(c)}{T_{ig}(c,c)(1 - \tau_{ig}^w(c)) w_i(c) s_i(c) \phi_i(c)} \right]^\theta
\] (42)

Parameters on wage and schooling time have been calibrated from the previous section. We assume a probability of working at young, \(P_{\text{work}}^ig\), are the same across occupations. So, the probability of working at young is the aggregate labor force participation rate of the young.

Given estimated parameters, we now move to the next year, 1970. We infer a friction in the next year \(\tau_{ig}^w(c + 1)\) by using the wage growth rate:

\[
\frac{\text{wage}_{ig}(c, c + 1)}{\text{wage}_{ig}(c,c)} = \frac{T_{ig}(c,c + 1)}{T_{ig}(c,c)} \cdot \left( \frac{P_{\text{work}}^ig(c,c + 1)}{P_{\text{work}}^ig(c,c)} \right)^{-\frac{1}{1-\eta}} \cdot \frac{w_i(c + 1)}{w_i(c)} \cdot \frac{s_i(c) \phi_i(c + 1)}{s_i(c) \phi_i(c)} \cdot \frac{1 - \tau_{ig}^w(c + 1)}{1 - \tau_{ig}^w(c)}
\] (43)

The first component is the occupation-specific productivity growth over age. We calibrate this term from white men's wage growth. However, if labor force participation of under-represented group has increased over time, productivity growth from the white men would overstate their experience accumulation. We adjust productivity growth by the change in labor force participation rate over life-cycle.

The last component is the change in the friction. We naturally bound the friction in 1970 between zero and maximum value from the aggregate friction of the young in 1970.

Given the friction in the labor market, a friction in schooling is easily calculated by the following equation:

\[
\tau_{ig}(c + 1) = \left( 1 + \tau_{ig}^h(c + 1) \right)^{\eta}
\] (44)

Given the friction in 1970, we can infer \(m_g, z_{ig}, \) and \(T_{\text{home}}^ig\) in 1970 from data on the labor force participation rate, occupational propensities, and average wages as before. Then, we estimate the friction in 1980 to match wage growth rate. By solving the prob-
lem sequentially until 2007, we obtain frictions in the labor market and human capital accumulation for every cohorts and time. Again, $\tau_{ig}^w$ is time-specific and $\tau_{ig}^h$ is cohort-specific as explained before.

We have two missing cohorts, who were the middle-aged and the old in 1960. We treat them as the young and infer $m_g$, $z_{ig}$, and $T_{ig}^{\text{home}}$.

**B3. Rent Transfer**

Given estimated frictions, we calculate total rents from the labor market and schooling and estimate equivalent subsidy rate to white men, balancing out a deadweight loss by the frictions. We do so by the following steps:

1. Calculate total rents from the labor market for all three groups, cohorts, time, and occupations:

$$ \text{rent}^{w}_g(t) = \sum_i \left\{ \tau_{ig}^w(t) \left( \sum_c H_{ig}(c,t) \right) w_i(t) \right\} $$

2. Calculate total rents from schooling for all three groups, young cohort, time, and occupations:

$$ \text{rent}^{h}_g(t) = \sum_i \left\{ \eta_i \frac{\tau_{ig}^h(t)}{1 + \tau_{ig}^w(t)} (1 - \tau_{ig}^w(t)) w_i(t) H_{ig}^{\text{young}}(t) \right\} $$

3. Calculate equivalent subsidy rate to white men so that total subsidy to the white men balances out total rents from other three groups. We apply population share by demographic groups in this calculation.
B4. Wage for Home Sector

We have estimated wage and schooling parameters, $w_i$ and $s_i$, as if we knew the subsidy rates to white men. We repeat the whole process until implicit wage for home sector, $w_{\text{home}}$, converges.